1 Introduction

1-1 Surveying

The purpose of surveying is to locate the positions of points on or near the surface of the earth. Some surveys involve the measurement of distances and angles for the following reasons: (1) to determine horizontal positions of arbitrary points on the earth's surface, (2) to determine elevations of arbitrary points above or below a reference surface, such as mean sea level, (3) to determine the configuration of the ground, (4) to determine the directions of lines, (5) to determine the lengths of lines, (6) to determine the positions of boundary lines, and (7) to determine the areas of tracts bounded by given lines. Such measurements are data-gathering measurements.

In other surveys it is required to lay off distances, angles, and grade lines to locate construction lines for buildings, bridges, highways, and other engineering works, and to establish the positions of boundary lines on the ground. These distances and angles constitute layout measurements.

A survey made to establish the horizontal or vertical positions of arbitrary points is known as a *control survey*. A survey made to determine the lengths and directions of boundary lines and the area of the tract bounded by these lines, or a survey made to establish the positions of boundary lines on the ground, is termed a *cadastral, land, boundary,* or *property survey*. A survey conducted to determine the configuration of the ground is termed a *topographic survey*. The determination of the configuration of the bottom of a body of water is a *hydrographic survey*. Surveys executed to locate or lay out engineering works are known as *construction surveys*. A survey performed by means of aerial photography is called an *aerial survey* or a *photogrammetric survey*.

The successful execution of a survey depends on surveying instruments of a rather high degree of precision and refinement, and also on the proper use and handling of these instruments in the field. All surveys involve some computations, which may be made directly in the field, or performed in the office, or in both places.

Some types of surveys require very few computations, whereas others involve lengthy and tedious computations. In the study of surveying therefore, the student not only must become familiar with the field operation techniques but must also learn the mathematics applied in surveying computations.

1-2 Basic Definitions

To gain a clear understanding of the procedures for making surveying measurements on the earth's surface, it is necessary to be familiar with the meanings of certain basic terms. The terms discussed here have reference to the actual figure of the earth.

An *oblate spheroid*, also called an *ellipsoid of revolution*, is a solid obtained by rotating an ellipse on (or around) its shorter axis. It is helpful to construct an idealized figure of the earth, which is usually an oblate spheroid where the earth's rotational axis serves as the shorter or minor axis. Because of its relief, the earth's surface is not a true spheroid. However, an imaginary surface representing a mean sea level extending over its entire surface very nearly approximates a spheroid. This imaginary surface is used as the figure on which surveys of large extent are computed.

A *vertical line* at any point on the earth's surface is the line that follows the direction of gravity at that point. It is the direction that a string will assume if a weight is attached to the string and the string is suspended freely at the point. At a given point there is only one vertical line. The earth's center of gravity cannot be considered to be located at its geometric center, because vertical lines passing through several different points on the surface of the earth do not intersect in that point. In fact, all vertical lines do not intersect in any common point. A vertical line is not necessarily normal to the surface of the earth, nor even to the idealized spheroid. The angle between the vertical line and the normal to the spheroid at a point is called the *deflection of the vertical*.

A *horizontal line* at a point is any line that is perpendicular to the vertical line at the point. At any point there are an unlimited number of horizontal lines.

A *horizontal plane* at a point is the plane that is perpendicular to the vertical line at the point. There is only one horizontal plane through a given point.

A *vertical plane* at a point is any plane that contains the vertical line at the point. There are an unlimited number of vertical planes at a given point.

A *level surface* is a continuous surface that is at all points perpendicular to the direction of gravity. It is exemplified by the surface of a large body of water at complete rest (unaffected by tidal action).

A *horizontal distance* between two given points is the distance between the points projected onto a horizontal plane. The horizontal plane, however, can be defined at only one point. For a survey the reference point may be taken as any one of the several points of the survey.

A *horizontal angle* is an angle measured in a horizontal plane between two vertical planes. In surveying this definition is effective only at the point at which the measurement is made or at any point vertically above or below it.

A *vertical angle* is an angle measured in a vertical plane. By convention, if the angle is measured upward from a horizontal line or plane, it is referred to as a *plus* or *positive* vertical angle, and also as an *elevation* angle. If the angle is measured downward, it is referred to as a *minus* or *negative* vertical angle, and also as a *depression* angle.

A *zenith angle* is also an angle measured in a vertical plane, except that, unlike a vertical angle, the zenith angle is measured down from the upward direction of the plumb line. Obviously the zenith angle is equal to 90° minus the vertical angle.

The *elevation* of a point is its vertical distance above or below a given reference level surface (see Section 3-1).

The *difference in elevation* between two points is the vertical distance between the two level surfaces containing the two points.

Plane surveying is that branch of surveying wherein all distances and horizontal angles are assumed to be projected onto one horizontal plane. A single reference plane may be selected for a survey where the survey is of limited extent. For the most part this book deals with plane surveying.

Geodetic surveying (sometimes referred to as *control surveying*) is that branch of surveying wherein all distances and horizontal angles are projected onto the surface of the reference spheroid that represents mean sea level on the earth.

The surveying operation of leveling takes into account the curvature of the spheroidal surface in both plane and geodetic surveying. The leveling operation determines vertical distances and hence elevations and also differences of elevation.

1-3 Units of Measurement

In the United States the linear unit most commonly used at the present time is the foot, and the unit of area is the acre, which is 43,560 ft^2. In most other countries throughout the world, distances are expressed in meters. The meter is also used by the National Geodetic Survey of the United States Department of Commerce as well as other federal and state agencies in the United States engaged in establishing control. However, the published results of some of these control survey operations are given in both units, or are available to the user in both units.

On all U.S. government land surveys, the unit of length is the Gunter's chain, which is 66 ft long and is divided into 100 links, each of which is 0.66 ft or 7.92 in. long. A chain, therefore, equals $\frac{1}{80}$ mile. This is a convenient unit where areas are to be expressed in acres, since 1 acre = 10 square chains. A distance of 2 chains 18 links can also be written as 2.18 chains. Any distance in chains can be readily converted into feet, if desired, by multiplying by 66.

In the Southwest portions of the United States that were influenced by the Spanish, another unit, known as the vara, has been used. A vara is about 33 in. long. The exact length varies slightly in different sections of the Southwest, where the lengths of property boundaries are frequently expressed in this unit.

For purposes of computation and plotting, decimal subdivisions of linear units are the most convenient. Most linear distances are therefore expressed in feet and tenths, hundredths, and thousandths of a foot. The principal exception to this practice is in the layout work on a construction job, where the plans of the structures are dimensioned in feet and inches. Tapes are obtainable graduated either decimally or in feet and inches.

Volumes are expressed in either cubic feet or cubic yards.

Angles are measured in degrees (°), minutes (′), and seconds (″). One circumference = 360°; 1° = 60′; 1′ = 60″. In astronomical work some angles are expressed in hours (^{h}), minutes (^{m}), and seconds (^{s}). Since one circumference = $24^h = 360°$, it follows that $1^h = 15°$ and $1° = \frac{1}{15}^h = 4^m$ (see Section 12-5).

Although some surveying instruments that measure angles in the sexagesimal system are graduated in degrees, minutes, and seconds, it is usually necessary, in computations with hand calculators, to convert to degrees and decimal degrees (unless the calculator has provision for this conversion) to compute the trigonometric functions of the angles, and then if necessary to convert back to degrees, minutes, and seconds. The number of decimal places to be retained in the decimal degree is a function of the least reading of the instrument or of the given angle. Equivalents of the decimal part of a degree are

$$
\begin{aligned}
0.00001° &= 0.036''\\
0.0001° &= 0.36''\\
0.001° &= 3.6''\\
0.01° &= 36''\\
0.1° &= 6'
\end{aligned}
$$

Thus, if the angle is given to the nearest minute, two decimal places must be retained; if the angle is given to the nearest second, four decimal places must be retained.

EXAMPLE 1-1 Convert 153° 43′ 17.2″ to decimal form.

Solution:

$$153°\ 43'\ 17.2'' = 153°\left(43\frac{17.2}{60}\right)' = 153°\ 43.287'$$

$$153°\ 43.287' = \left(153\frac{43.287}{60}\right)° = 153.72144°$$

which is also written 153°.72144.

EXAMPLE 1-2 Convert 24.4652° to degrees, minutes, and seconds.*

Solution:

$$0.4652° \times 60 = 27.912'$$

$$0.912' \times 60 = 54.7''$$

The value of the conversion is thus 24° 27′ 54.7″

*Many handheld calculators have provision for converting from degrees, minutes, and seconds to decimal degrees and vice versa using conversion keystrokes.

To help visualize the physical size of small dimensions, consider that 0.01 ft is very nearly equal to $\frac{1}{8}$ in., and that 0.10 ft is therefore about $1\frac{1}{4}$ in. Also, the sine or tangent of 1′ of arc is approximately 0.0003 (three zeroes 3). Thus 1′ of arc subtends about 0.03 ft or $\frac{3}{8}$ in. in 100 ft. At 1000 ft, 1′ of arc subtends 0.30 or $3\frac{1}{2}$ in.

The sine or tangent of 1″ of arc is approximately 0.000005 (five zeroes 5). Thus at 1000 ft, 1″ subtends 0.005 ft or about $\frac{1}{16}$ in. Another relationship is embodied in the expression "a second is a foot in 40 miles." Taking the radius of the earth to be 4000 miles, a second of arc at the center of the earth subtends approximately 100 ft on the earth's surface.

Quite frequently it becomes necessary to convert a small angle from its arc or radian value to its value expressed in seconds, and vice versa. A unit radian is the angle subtended by an arc of a circle having a length equal to the circle's radius. Thus 2π rad = 360°; 1 rad = 57° 17′ 44.8″ and 0.01745 rad = 1°. We find from trigonometry that the sine, the tangent, and the arc or radian value of 1″ are all equal within the limits of computational practicality. As a consequence of this, and designating θ as a small angle,

$$\text{arc } \theta \approx \sin \theta \approx \tan \theta$$

If the small angle is expressed in seconds, its arc, sine, or tangent can be determined by the following relationships:

$$\begin{aligned} \text{arc } \theta &= \theta'' \times \text{arc } 1'' && \text{(exact)} \\ \sin \theta &= \theta'' \times \sin 1'' && \text{(approximate)} \\ \tan \theta &= \theta'' \times \tan 1'' && \text{(approximate)} \end{aligned}$$

The value of the arc, sine, and tangent of 1″ is 0.0000048481 to 10 decimal places. If the arc or radian value of a small angle is known, or if the sine or tangent is known, then its value in seconds can be determined by rearranging the three preceding expressions to give

$$\theta'' = \frac{\text{arc } \theta}{\text{arc } 1''} = \frac{\text{arc } \theta}{48,481 \times 10^{-10}} = 206,265 \text{ arc } \theta$$

$$\theta'' = \frac{\sin \theta}{\sin 1''} = \frac{\tan \theta}{\tan 1''}$$

EXAMPLE 1-3 A target 4 in. wide is placed on the side of a building 1600 ft away from a survey point. What angle is subtended at the point between the left and right edges of the target?

Solution: The target width is 0.333 ft and thus the arc of the small angle is 0.333/1600 = 0.000208. The angle in seconds is then 0.000208/48,481 × 10^{-10} = 42.9″ or 0.000208 × 206,265 = 42.9″

- When the digit to be dropped is less than 5, the number is written without the digit. Therefore 117.573 becomes 117.57.
- When the digit to be dropped is exactly 5, the nearest even number is used. Therefore 117.575 becomes 117.58 and 117.565 becomes 117.56.
- When the digit to be dropped is more than 5, the preceding digit is increased by 1. Therefore 117.578 becomes 117.58.

When performing computations it is important to use an appropriate number of significant figures so that the computations themselves do not lessen the accuracy of the measurements and consequently the result. Likewise, it is misleading to carry more digits than is warranted. For example, if the area of a parcel is created by adding three separate smaller parcels together, with each sub-area having been determined by a different surveying technique, the result can be stated in a misleading fashion as shown:

Parcel A	3.684	acres
Parcel B	6.01	acres
Parcel C	11.0	acres
Total	= 20.694	acres
Correct answer	= 20.7	acres

The accuracy of the resulting area is probably no better than ±0.05 acres, because Parcel C's area was determined only to this accuracy.

Likewise, when multiplying and dividing, the number of significant figures in the answer is equal to the least number of significant figures in the terms of the product or quotient. For example, $117.58 \times 6.1 = 717.24$, but the answer should be shown as 720 unless both quantities were stated as exact numbers.

It is intuitive that when using a calculator or computer, round-off error occurs. Therefore it is advisable to use more decimal places in the computations than are significant. The number of places required is a function of the number of computations, but in plane surveying eight places will usually suffice. As with any such "rule," there are numerous exceptions.

1-10 Errors and Mistakes

The value of a distance or an angle obtained by field measurements is never exactly the true value, except by chance. The measured value approaches the true value as the number and size of errors in the measurements become increasingly small. An error is the difference between the true value of a quantity and the measured value of the same quantity. Errors result from instrumental imperfections, personal limitations, and natural conditions affecting the measurements. Examples of instrumental errors are (1) a tape that is actually longer or shorter than its indicated length; (2) errors in the graduations of the circles of an engineer's transit; and (3) a defect in the calibration of an electronic distance measuring device. Examples of personal limitations are the observer's inability to bisect a target or read a vernier exactly, inability to maintain a steady tension on the end of a tape, and failure to keep a level bubble centered at the instant at which a leveling observation is taken. Examples of natural conditions affecting a measurement are wind and temperature, and pressure and humidity changes causing the refraction of the direction or distance between two points.

An error is either a *systematic error* or a *random error*. A systematic error is one for which the magnitude and algebraic sign can theoretically be determined. If a tape is found to measure 99.94 ft between the 0-ft mark and the 100-ft mark when compared with a standard, then the full tape length introduces a systematic error of +0.06 ft each time it is used to measure the distance between two given points. If a tape is used at a temperature other than that at which it was compared with a standard, then the amount by which the nonstandard temperature increases or decreases the length of the tape can be computed from known characteristics of the material of which the tape is made.

A random error is one for which the magnitude and sign cannot be predicted. It can be plus or minus. Random errors tend to be small and tend to distribute themselves equally on both sides of zero. If an observer reads and records a value of, say, 6.242 ft when the better value is 6.243 ft, a random error of − 0.001 ft has been introduced. When an individual is holding a signal on which an instrument man is sighting, failure to hold the signal directly over the proper point will cause a random error of unknown size and algebraic sign in the measured angle. If, however, he *fixes* the signal eccentrically, the resultant error will be systematic.

Random errors tend to grow proportional to the square root of the number of them, but systematic errors grow directly proportional to the number of them.

A *mistake* is not an error but is a blunder on the part of the observer. Examples of mistakes are failure to record each full tape length in taping, misreading a tape, interchanging figures, and forgetting to level an instrument before taking an observation. Mistakes are avoided by exercising care in making measurements, by checking readings, by making check measurements, and to a great extent by common sense and judgment. If, for example, a leveling rod is read and the reading is recorded as 7.13 ft, whereas the levelman knows that this is absurd since he is very nearly at the top of a 14-ft rod, then he is exercising common sense in suspecting a mistake.

The subject of random errors is considered in more detail in Chapter 5. Systematic errors and methods for their elimination are discussed in the appropriate sections throughout the book.

1-11 Accuracy and Precision

Since surveying is after all a measurement science, it is necessary to distinguish between the two terms *accuracy* and *precision* that, if not understood, can cause needless confusion. The accuracy of a measurement is an indication of how close it is to the true value of the quantity that has been measured. To obtain an accurate measurement, the measuring instrument must have been calibrated by comparison with a standard. This allows for the elimination of systematic errors.

The precision of a measurement has to do with the refinement used in taking the measurement, the quality (but not necessarily the accuracy) of an instrument, the repeatability of the measurement, and the finest or least count of the measuring device.

As an illustration of the difference between these two terms, suppose that two different taping parties (see Chapter 2) measure the same line five times, each using a different 100-ft tape. The first party reports the following measurements: 736.80, 736.70, 736.75, 736.85, and 736.65 ft. The second party reports the following measurements: 736.42, 736.40, 736.40, 736.42, and 736.41 ft. Further suppose that the correct or true length of the line is 736.72 ft. Obviously, from an examination of the spread of the results, the measurements reported by the second party are more precise. However, those reported by the first party are more accurate because they tend to group around the true value. Thus the tape used by the second party has some kind of systematic error that has not been accounted for in the reported measurements.

PROBLEMS

1-1. How many hectares are contained in a rectangular field that measures 352.25 by 196.80 m?

1-2. How many acres are contained in the area given in Problem 1-1?

1-3. How many square feet are contained in the area given in Problem 1-1?

1-4. A distance of 40 chains 2 links is shown on a map between two boundary markers. What is the corresponding length in feet?

1-5. What is the length of the line in Problem 1-4 in meters?

1-6. Based on the equivalence 1 U.S. foot = 30.48 cm, how many feet are contained in a line that measures 27,542.331 m?

1-7. How many survey feet are contained in the line of Problem 1-6?

1-8. Convert the following decimal degrees to their corresponding values in the sexagesimal system: **(a)** 26.9°, **(b)** 196.23°, **(c)** 63.464°, **(d)** 312.1546°, **(e)** 19.52496°.

1-9. Convert the angles of Problem 1-8 to their equivalent grad values.

1-10. Convert the following angles to decimal degree form: **(a)** 16° 37′, **(b)** 254° 16′ 42″, **(c)** 96° 52′ 14.3″, **(d)** 35° 47′ 16.82″, **(e)** 174° 49′ 16.834″.

1-11. Convert the angles given in Problem 1-10 to their grad values.

1-12. Convert the following angles to decimal degrees: **(a)** 16^g, **(b)** 26.2^g, **(c)** 274.56^g, **(d)** 394.163^g, **(e)** 27.9444^g, **(f)** 37.46205^g.

1-13. Convert the angles given in Problem 1-12 to their values in the sexagesimal system.

1-14. Convert the following volumes to cubic meters: **(a)** 14,955 yd^3, **(b)** 129,590 yd^3, **(c)** 2545.6 yd^3, **(d)** 18,192.66 yd^3.

1-15. Convert the following volumes to cubic yards: **(a)** 546 m^3, **(b)** 2850.2 m^3, **(c)** 1485.66 m^3, **(d)** 42,455 m^3.

1-16. The sides of a triangle measure 1040.25, 1855.24, and 1318.18 m. Compute the three angles in the triangle expressed to the nearest 0.0001^g.

1-17. The sides of a triangle measure 302.55, 1696.20, and 1714.36 ft. Compute the three angles in the triangle (nearest second).

1-18. What is the area of the triangle of Problem 1-16 in hectares?

1-19. What is the area of the triangle of Problem 1-17 in acres?

1-20. Two sides and the included angle of a triangle are 1810.462 m, 1462.82 m, and 35.6362^g, respectively. Compute the length of the remaining side (nearest millimeter) and the two remaining angles (nearest 0.0001^g).

1-21. Two sides and the included angle of a triangle are 545.156 m, 1395.832 m, and 15° 22′ 46″, respectively. Compute the length of the remaining side (nearest millimeter) and the remaining two angles (nearest second).

1-22. Compute the area of the triangle in Problem 1-20 in hectares and in acres.

1-23. Compute the area of the triangle in Problem 1-21 in hectares and in acres.

1-24. In triangle ABC, $A = 27° 14' 52''$, $B = 52° 35' 44''$, and side $AB = c = 385.462$ ft. Compute sides a and b (nearest 0.001 ft).

1-25. The radius of a circle is 350.000 m. What arc length is subtended by a central angle of 125.4652^g (nearest millimeter).

1-26. The arc of a small angle is 0.0000621458. What is the angle in seconds (nearest 0.01″)?

1-27. What is the angle in Problem 1-26 in grads (nearest 0.00001^g)?

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to hold the tape, a taping pin can be slipped through the eye at the end of the tape and used as a handle. A tape that is thrown together in the form of a series of loops when not in use must be carefully unwrapped and checked for short kinks before it can be used for measurement. As long as a tape is stretched straight, it will stand any amount of tension that two people can apply. If kinked or looped, however, a very slight pull is sufficient to break it.

2-9 Measurements with Tape Horizontal

The horizontal distance between two points can be obtained with a tape either by keeping the tape horizontal or by measuring along the sloping ground and computing the horizontal distance. For extreme precision, such as is required in determining the length of a baseline in a triangulation system, the latter method is used. This method is also advantageous where steep slopes are encountered and it would be difficult to obtain the horizontal distance directly.

For moderate precision where the ground is level and fairly smooth, the tape can be stretched directly on the ground, and the ends of the tape lengths can be marked by taping pins or by scratches on a paved area. Where the ground is level but ground cover prevents laying the tape directly on the ground, both ends of the tape are held at the same distance above the ground by the forward tapeman and the rear tapeman. The tape is preferably held somewhere between knee height and waist height. The graduations on the tape are projected to the ground by means of the plumb bobs. The plumb-bob string is best held on the tape graduation by clamping it with the thumb, so that the length of the string can be altered easily if necessary (this can be seen in Fig. 2-4). When a tape is supported throughout its length on the ground and subjected to a given tension, a different value for the length of a line will be obtained than when the tape is supported only at the two ends and subjected to the same tension (see Section 2-18). Where fairly high accuracy is to be obtained, the method of support must be recorded in the field notes, provided different methods of support are used on one survey. Experienced tapemen generally obtain equivalent results by plumbing the ends of the tape over the marks or by having the tape supported on the ground.

When the ground is not level, either of two methods may be used. The first is to hold one end of the tape on the ground at the higher point, to raise the other end of the tape until it is level, either by estimation or with the aid of the hand level, and then to project the tape graduation over the lower point to the ground by means of a plumb bob. The other method is to measure directly on the slope as described in Sections 2-11 and 2-12. These methods are shown in Fig. 2-2.

For high precision, a taping tripod or taping buck must be used instead of a plumb bob. Such a tripod is shown in Fig. 2-3. Taping tripods are usually used in groups of three, the rear tripod then being carried to the forward position. A pencil mark is scribed at the forward tape graduation, and on the subsequent measurement the rear tape graduation is lined up with this mark in order to carry the measurement forward. Since taping is usually done on the slope when tripods are used, the elevations of the tops of the tripods must be determined at the same time the taping proceeds. The elevations, which are determined by leveling (Chap-

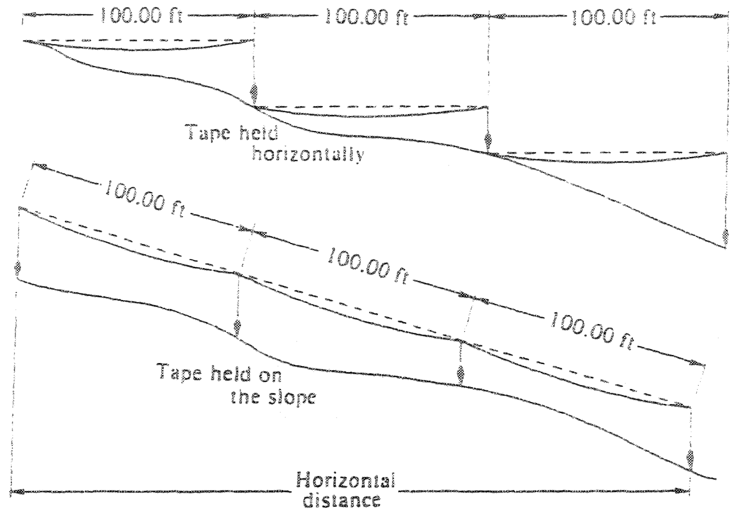

FIGURE 2-2 Taping over sloping ground using 100-ft tape.

FIGURE 2-3 Taping tripod.

ter 3), give the data necessary to reduce the slope distances to horizontal distances as discussed in Section 2-12.

The head tapeman carries the *zero* end of the tape and proceeds toward the far end of the line, stopping at a point approximately a tape length from the point of beginning. The rear tapeman lines in the forward end of the tape by sighting on the line rod at the far end of the line. Hand signals are used to bring the head tapeman on line. The rear tapeman takes a firm stance and holds the tape close to his body with one hand, either wrapping the thong around his hand as shown in Fig. 2-4 or holding a taping pin that has been slipped through the eye of the tape. Standing

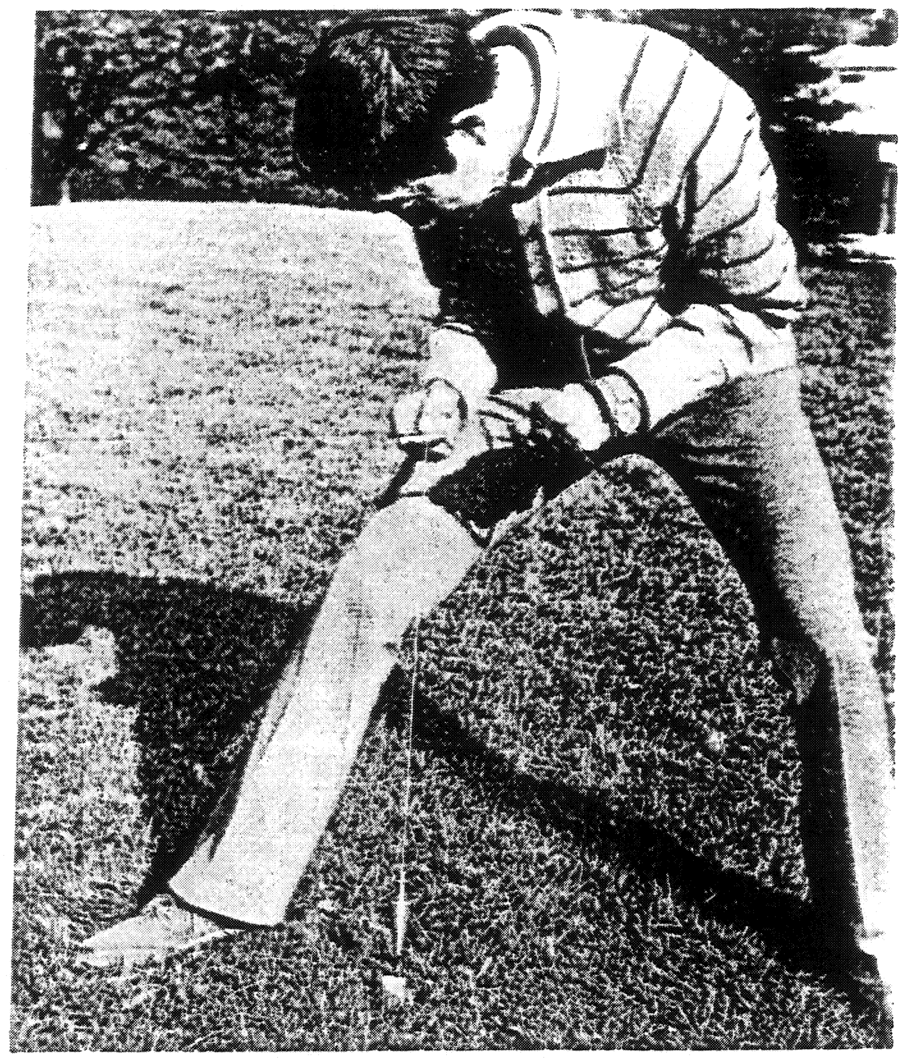

FIGURE 2-4 Plumbing over point.

to the side of the tape, he plumbs the end graduation over the point on the ground marking the start of the line. The tip of the plumb bob should be less than $\frac{1}{8}$ in. (about 3 mm) above the ground point.

The head tapeman applies the tension to be used, either by estimation or by means of a spring balance fastened to the zero or forward end of the tape. At approximately the correct position on the ground, he clears a small area where the taping pin will be set. After again applying the tension, the head tapeman waits for a vocal signal from the rear tapeman, indicating that the latter is on the rear point. When the plumb bob has steadied and its tip is less than $\frac{1}{4}$ in. or about 5 mm from the ground, the head tapeman dips the end of the tape slightly so that the plumb bob touches the ground. Then he, or a third member of the taping party, sets a taping pin or a surveying tack at the tip of the plumb bob to mark the end of the first full tape length, as shown in Fig. 2-5. The pin is set at right angles to the line and inclined at an angle of about 45° with the ground away from the side on which the rear tapeman will stand for the next measurement. The tape is then stretched out again to check the position of the pin. The notekeeper records the distance, 100.00 ft, or 30.000 m, in the field notes. The tape is advanced another tape length, and the entire process is repeated.

If the taping advances generally downhill, the head tapeman checks to see that the tape is horizontal by means of the hand level. If the taping advances generally uphill, the rear tapeman checks for level.

When the end of the line is reached, the distance between the last pin and the point at the end of the line will usually be a fractional part of a tape length. The rear tapeman holds the particular full-foot or decimeter graduation that will bring the subgraduations at the zero end of the tape over the point marking the end of the line. The head tapeman rolls the plumb-bob string along the subgraduations with his thumb until the tip of the plumb bob is directly over the ground point marking the end of the line.

Two types of end graduations of a tape that reads in feet are shown in Fig. 2-6. In view (a) the subgraduations are outside the zero mark, and the fractional part of a foot is added to the full number of feet. Hence the distance is 54 + 0.46 = 54.46 ft. This type is referred to as an *add* tape. In view (b) the subgraduations are between the zero and the 1-ft graduation, and the fractional part of a foot must be subtracted from the full number of feet. So the distance is 54 − 0.28 = 53.72 ft. This type is called a *cut* tape. Because of the variation in the type of end graduations, the rear tapeman must call out the actual foot mark he holds, and both the head tapeman and the notekeeper must agree that the value recorded in the field notes is the correct value.

Add tapes are more convenient to use than cut tapes simply because it is easier to add than to subtract the decimal part of the whole unit. The design of an add metric tape that is divided in decimeters throughout its length is shown in Fig. 2-7. The outside decimeter is further subdivided to centimeters and then to millimeters. In the illustration the rear tapeman holds the 14.7-m mark at the pin, and the head tapeman reads 0.072 m at the end of the line. The distance is thus 14.7 + 0.072 = 14.772 m.

FIGURE 2-5 Setting taping pin to mark forward position of tape.

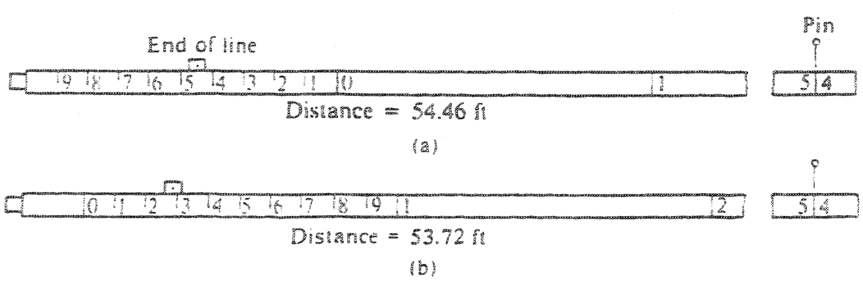

FIGURE 2-6 Graduations at end of foot-graduated tape

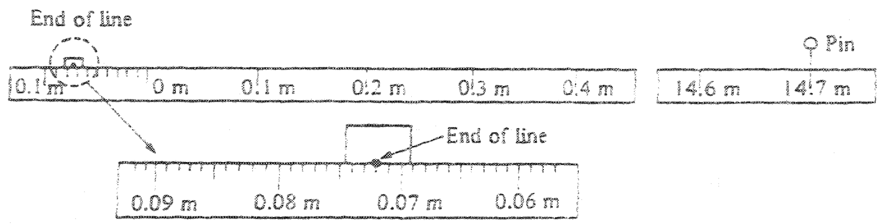

FIGURE 2-7 Graduations at end of metric tape.

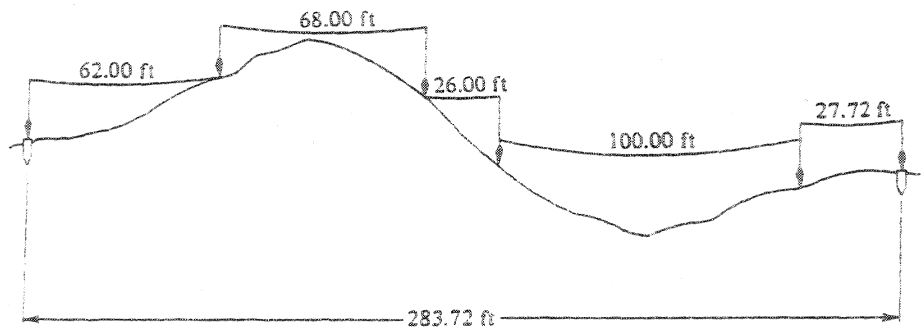

FIGURE 2-8 Breaking tape.

Where the slope is too steep to permit bringing the full length of the tape horizontal, the distance must be measured in partial tape lengths, as shown in Fig. 2-8. It is then necessary to enter a series of distances in the field notes. Some or all of them will be less than a full tape length. For a partial tape length, the head tapeman holds the zero end and the rear tapeman holds a convenient whole foot or decimeter mark that allows the selected length of tape to be horizontal. When the forward pin is set, this partial tape length is recorded in the field notes. The head tapeman then advances with the zero end of the tape, and the rear tapeman again picks up a convenient whole foot or decimeter mark and plumbs it over the pin. Each partial tape length is recorded as it is measured or as the forward pin is set. Fig. 2-9 illustrates the use of a device called a *tape clamp* for holding a tape at any place other than at an end.

If a tape clamp is not available, the rear tapeman must hold the tape in one hand in such a manner that it neither injures his hand nor damages the tape. At the same time he must be able to sustain a tension of between 10 and 20 lb, or between 5 and 10 kg. The technique shown in Fig. 2-10 is a satisfactory solution to this problem. The tape is held between the fleshy portion of the fingers and that of the palm. Enough friction is developed to sustain a tension upward of 25 to 30 lb (10 to 15 kg) without injury or discomfort to the tapeman. He must not turn his hand too sharply, however, otherwise the tape may become kinked.

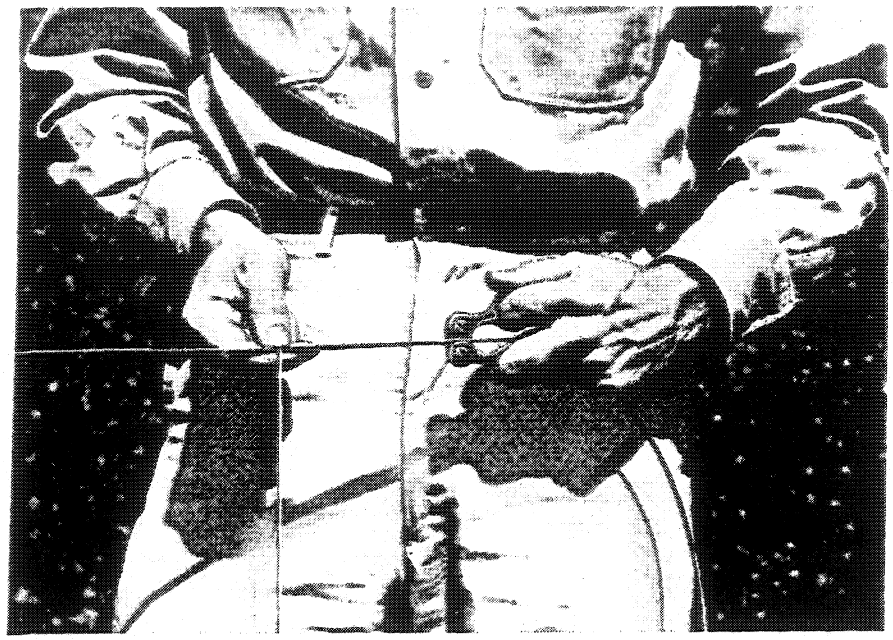

FIGURE 2-9 Use of tape clamp.

All distances should be taped both forward and backward, to obtain a better value of the length of the line and to detect or avoid mistakes. When the backward measurement is made, the new positions of the pins should be completely independent of their previous positions. This practice eliminates the chance of repeating a mistake.

2-10 Tension

Most steel tapes are correct in length at a temperature of 68°F (20°C) when a tension of 10 to 12 lb or 5 kg is used and the tape is supported throughout the entire length. If this same tension is used when the tape is suspended from the two ends, the horizontal distance between the ends of the tape will be shorter than the nominal length. The amount of the shortening depends on the length and the weight of the tape. A light 100-ft tape weighs about 1 lb. Such a tape, when suspended from the two ends, would be shortened about 0.042 ft under a tension of 10 lb. A heavy 100-ft tape weighs about 3 lb and would be shortened about 0.375 ft under a tension of 10 lb. Some engineers attempt to eliminate this error by increasing the tension used. The tension for the light tape is then increased to about 18 lb. It is practically impossible to eliminate the error in the heavy tape by this method, as the tension would have to be increased to about 50 lb. Generally, less tension is used and a correction is applied to each measured length.

2-13 Systematic Errors in Taping

The principal systematic errors in linear measurements made with a tape are (1) incorrect length of tape, (2) tape not horizontal, (3) fluctuations in the temperature of the tape, (4) incorrect tension or pull, (5) sag in the tape, (6) incorrect alignment, and (7) tape not straight.

2-14 Incorrect Length of Tape

At the time of its manufacture, a tape is graduated when under a tension of about 10 lb or 4.5 kg. A steel tape will maintain a constant length under a considerable amount of handling and abuse. This is not true, however, of an invar tape, which must be handled with great care. In either case, if a tape is compared with a standard length under specific conditions of temperature, tension, and method of support, the distance between the two end graduations will seldom equal the nominal length indicated by the graduation numbers. The correction to be applied to any measurement made with the tape to account for this discrepancy is called the *absolute correction* C_a, and is given by

$$C_a = \text{true length} - \text{nominal length} \tag{2-8}$$

The true length is the value determined by calibration under specific conditions. The calibration or standard tension will range anywhere from 10 to 30 lb or 5 to 15 kg and is specified by the user of the tape. The calibration comparison can be made with the tape supported throughout its length, or supported only at the two ends, or supported at the two ends and at one or more intermediate points.

The absolute error $-C_a$ is usually assumed to be distributed uniformly throughout the length of the tape. Thus the absolute correction in a measured distance is directly proportional to the number and fractional parts of the tape used in making the measurement.

2-15 Tape Not Horizontal

If the tape is assumed to be horizontal but actually is inclined, an error is introduced. The amount of this error C can be computed from Eq. (2-6) or may be taken as $h^2/2s$. If one end of a 100-ft tape is 1.41 ft higher or lower than the other, the error will amount to 0.01 ft. It should be noted that the error is proportional to the square of the vertical distance. When one end is 2.82 ft above or below the other, the error increases to 0.04 ft.

Errors from this source are cumulative and may be considerable when measuring over hilly ground. The error can be kept at a minimum by using a hand level to determine when the tape is horizontal.

2-31 Accuracy of EDM Measurements

At close range the accuracy of the EDM is limited by a constant uncertainty such as 1 cm, 5 mm, or 0.01 ft. As the measured distance is increased, this constant value becomes relatively inconsequential. Beyond, say, 500 to 1000 m, all of the currently available EDMs will give relative accuracies of better than 1 part in 25,000. This is very difficult to obtain by taping and requires taping tripods and very careful attention to systematic errors. On the other hand this accuracy is practically assured using the EDMs. The factors that limit both the relative and the absolute accuracies of EDM measurements are the meteorological conditions at the time of measurement. If these are known with sufficient accuracy, then all but the very short-range instruments are capable of relative accuracies of 1 part in 100,000 or better.

If very long lines must be measured with the maximum accuracy, then the uncertainty of the meteorological conditions along the entire beam path becomes important. For the majority of measurements in the short to intermediate range, meteorological measurements taken only at the instrument end of the line are sufficient to obtain the desired accuracy. This level of accuracy can be enhanced by taking a mean of the readings at both ends of the line. Improvements can further be made by sampling the meteorological conditions at intermediate points along the line, which of course is complicated usually by the necessity of elevating the meteorological instruments to considerable heights above the intervening terrain. The ultimate solution at present is to fly along the line and record the temperature and pressure profile all along the line. This technique has been employed in California along lines used to measure very small earthquake fault displacements over great distances. The various manufacturers usually state the accuracy as $\sigma = a + bd$, where d is the distance and a and b are constants for a particular instrument.

PROBLEMS

2-1. A surveyor paces a 100-ft length six times with the following results: $35\frac{1}{2}$, 34, $34\frac{3}{4}$, 35, $34\frac{3}{4}$, and 35 paces. How many paces must be stepped off to lay out a distance of 20 chains?

2-2. A surveyor paces a 50-m length five times with the following results: $56\frac{1}{2}$, 57, $56\frac{1}{2}$, 58, and 57 paces. How many paces must he step off to lay out a distance of 450.00 m?

2-3. A slope distance of 962.21 ft is measured between two points with a slope angle of 3° 16′. Compute the horizontal distance between the two points.

2-4. In Problem 2-3, if the vertical angle is in error by 2′, what error is produced in the horizontal distance?

2-5. A horizontal distance of 850.00 ft is to be laid out on a 2° 58′ slope. What slope distance must be laid out?

2-6. The difference in elevation between two points is 16.264 m. The measured slope distance is 343.516 m. Compute the horizontal distance.

2-7. A measurement of 148.264 m is made on a 4° 16′ slope. Compute the corresponding horizontal distance.

2-8. In problem 2-6, if the difference in elevation is 0.022 m, what error is produced in the horizontal distance?

2-9. A line was measured along sloping ground with a 30-m tape, and the following results were recorded:

Slope Distance (m)	Difference in Elevation (m)
30.000	1.792
30.000	0.930
18.520	0.966
30.000	3.075
12.422	0.660

What is the horizontal length of the line?

2-10. A tape that measures 99.96 ft between the zero and 100-ft mark is used to lay out foundation walls for a building 280.00 × 560.00 ft. What observed distances should be laid out?

2-11. A tape is calibrated and found to measure 100.04 ft between the 0- and 100-ft marks. What measurements should be laid out to establish a horizontal distance of 682.25 ft?

2-12. What distance on a 5% grade should be laid out with a tape that measures 30.010 m under field conditions if the horizontal distance is to be 430.000 m?

2-13. A 100-ft steel tape measures correctly when supported throughout its length under a tension of 10 lb and at a temperature of 72°F. It is used in the field at a tension of 18 lb and supported at the two ends only. The temperature throughout the measurement is 84°F. The measured length is 748.25 ft (the tape is suspended between the 48-ft mark and the zero end for the last measurement). The tape weighs 2.00 lb and has a cross-sectional area of 0.0060 in^2. Assuming that E is 28,000,000 psi for steel, what is the actual length of the line?

2-14. A 30-m tape weighs 12 g/m and has a cross section of 0.020 cm^2. It measures correctly when supported throughout under a tension of 8.5 kg and at a temperature of 20°C. When used in the field, the tape is supported at its two ends only, under a tension of 8.5 kg. The temperature is 13.5°C. What is the distance between the 0- and 30-m marks under these conditions?

2-15. A 100-ft tape is calibrated at 68°F and is found to measure 99.990 ft. A distance is measured as 515.68 ft at a temperature of 42°F. What is the correct distance?

2-16. A 100-ft steel tape weighs 1.80 lb and has a cross-sectional area of 0.0056 in^2. The tape measures 100.00 ft when supported throughout under a tension of 10 lb. Assume that $E = 28 \times 10^6$ psi. What tension, to the nearest $\frac{1}{4}$ lb, must be applied to overcome the effect of sag when the tape is supported at the two ends only?

2-17. A 50-m tape weighs 24 g/m and has a cross section of 0.038 cm^2. It measures 49.9862 m under a tension of 2.20 kg when supported at the two ends only. What does the tape measure if it is supported throughout under a tension of 6 kg? $E = 2.1 \times 10^6$ kg/cm^2.

2-18. With what accuracy must the difference in elevation between two ends of a 100-ft tape be known if the difference in elevation is 9.20 ft and the accuracy ratio is to be at least 1 : 10,000?

2-19. With what accuracy must a difference in elevation between two ends of a 30-m tape be known if the difference in elevation is 2.840 m and the accuracy ratio is to be at least 1 : 25,000?

2-20. The measured slope angle of a taped distance is 3° 54′. To what accuracy must the slope angle be measured if the relative accuracy is to be at least 1 : 20,000?

2-21. The measured slope angle of a measured slope distance of 342.535 m is 2° 24′. To what accuracy must the slope angle be measured if the horizontal distance is to be accurate to 5 mm?

2-22. Compute the refractive index of mercury vapor light at a temperature of 88°F and barometric pressure of 29.00 in.Hg. Neglect the effect of vapor pressure.

2-23. What is the refractive index of red laser light at a temperature of 20°C and barometric pressure of 725 mmHg? Neglect the effect of vapor pressure.

2-24. Microwaves are propagated through an atmosphere of 66°F, atmospheric pressure of 29.2 in.Hg, and vapor pressure of 0.51 in.Hg. If the modulating frequency is 30 MHz, what is the modulated wavelength?

2-25. What is the modulated wavelength of light of Problem 2-22 if the frequency of modulation is 30 MHz?

2-26. What is the modulated wavelength of light of Problem 2-23 if the frequency of modulation is 30 MHz?

2-27. Microwaves are modulated at a frequency of 75 MHz. They are propagated through an atmosphere at a temperature of 16°C, atmospheric pressure of 749 mmHg, and vapor pressure of 7.2 mmHg. What is the modulated wavelength?

2-28. Referring to Fig. 2-25, *AB* measures 796.16 ft; *BC* measures 423.25 ft; *AC* measures 1219.28 ft using a particular EDM reflector combination. A line measures 2946.22 ft with this instrument-reflector combination. What is the correct length of the line?

2-29. The height of an EDM set up at *A* is 5.32 ft. The height of the reflector set up at *B* is 4.30 ft. The height of the theodolite set up at *A* and used to measure a vertical angle is 5.00 ft. The height of the target at *B* on which the vertical angle sight is taken is 5.00 ft. The vertical angle is +4° 20′ 15″. The slope distance, after meteorological corrections, is 3451.55 ft. What is the horizontal distance between *A* and *B*?

2-30. The height of an EDM set up at *M* is 1.550 m. The height of the reflector set up at *P* is 1.400 m. The height of the theodolite set up at *M* used to measure the vertical angle is 1.600 m. The height of the target at *P* on which the vertical angle is sighted is 1.458 m. The slope distance, after meteorological corrections, is 975.26 m. The measured vertical angle is +3.2644^g. What is the horizontal distance between *M* and *N*?

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3 Leveling

3-1 *Introduction*

Leveling is the operation in surveying performed to determine and establish elevations of points, to determine differences in elevation between points, and to control grades in construction surveys. The elevation of a point has been defined as its vertical distance above or below a given reference level surface. The reference level surface used in the United States is the National Geodetic Vertical Datum of 1988. The NAVD 88 supersedes the National Geodetic Vertical Datum of 1929 (NGVD 29), which was the former official height reference (vertical datum) for the United States. NAVD 88 provides a modern, improved vertical datum for the United States, Canada, and Mexico. The NAVD 88 heights are the result of a mathematical (least squares) adjustment of the vertical control portion of the National Geodetic Reference System. Over 500,000 permanent benchmarks are included in the datum. The datum surface is an equipotential surface that passes through a point on the International Great Lakes Datum. The datum closely corresponds with mean sea level along the coasts of the United States.

A *benchmark* is a permanent or semipermanent physical mark of known elevation. It is set as a survey marker to provide a point of beginning for determining elevations of other points in a survey. A good benchmark is a bronze disk set either in the top of a concrete post or in the foundation of a structure. Specially designed marks used by the National Geodetic Survey (NGS) are the most stable. An NGS benchmark consists of a rod encased in grease inside a PVC pipe. Other locations for benchmarks are the top of a culvert headwall, the top of an anchor bolt, or the top of a spike driven into the base of a tree. The elevations of benchmarks are determined to varying degrees of accuracy by the field operations to be described in this chapter. Benchmarks established throughout the country by the NGS to a high order of accuracy define the North American Vertical Datum of 1988 (NAVD 88).

The basic instrument used in leveling is a spirit level that establishes a horizontal line of sight by means of a telescope fitted with a set of cross hairs and a level bubble. The level is described in later sections of this chapter. Other instruments used for determining vertical distances are the engineer's transit, the theodolite, the EDM, the aneroid barometer, the hand level, and the telescopic alidade. The use of the transit and the theodolite are explained in Chapters 4 and 14. A type of elevation can be determined by signals from the Global Positioning System (GPS) of satellites and will be discussed in Chapter 10.

3-2 Curvature and Refraction

The measurements involved in leveling take place in vertical planes. Consequently, the effect of earth curvature and atmospheric refraction must be taken into account because these effects occur in the vertical direction. They are allowed for either by the appropriate calculations or else by the measuring techniques designed to eliminate these effects.

To examine the effect of earth curvature, consider the amount by which the level surface passing through point A of Fig. 3-1 departs from a horizontal plane at the distance AB from the point of tangency. This departure is shown to be the vertical distance CB. If the effect of earth's curvature is designated c, and the distance from the point of tangency to the point in question is designated K, then the curvature effect can be shown to be

$$c = 0.667K^2 \tag{3-1}$$

in which c is the curvature effect in feet and K is the distance in miles. This expression for earth curvature is based on a mean radius of the earth, or 3959 miles. It should be noted that the error due to curvature is proportional to the square of the distance from the point of tangency to the point in question. Thus, for a distance of 10 miles, the value of c is about 67 ft, whereas for a distance 100 ft, it diminishes to 0.00024 ft. In the metric system

$$c_M = 0.0785K_M^2 \tag{3-2}$$

in which c_M is the curvature effect, in meters; K_M is the distance, in kilometers.

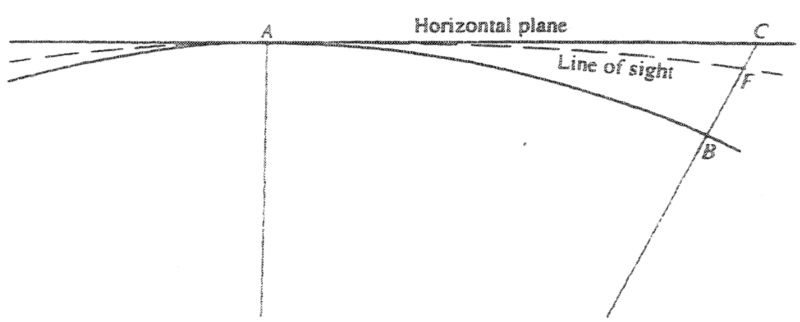


FIGURE 3-1 Curvature and refraction.

vertical distance CE is equal to $DC \sin \alpha$ or $DE \tan \alpha$, depending on whether the slope distance obtained by EDM or horizontal distance from triangulation (see Chapter 9) is known. The vertical distance from a level line through D to point C is FC, which is $CE + EF$. But EF is the total effect of curvature and refraction, which, by Eq. (3-4), is $0.0206(DE/1000)^2$ or $0.0206(DC \cos \alpha /1000)^2$. The difference in elevation between A and B is then $AD + EF + CE - CB$. The values of AD and CB are recorded at the time of measurement.

To eliminate the uncertainty in the curvature and refraction correction, vertical-angle observations are made at both ends of the line as close in point of time as possible. This pair of observations is termed *reciprocal vertical-angle observation.* In Fig. 3-3(b) the angle at K is assumed to be 90°. The vertical distance JK is equal to $HJ \sin \beta$ or $HK \tan \beta$. The vertical distance from a level line through H to point J is JL, which is $JK - KL$. But KL is the total effect of curvature and refraction, which is $0.0206\ (HK/1000)^2$ or $0.0206(HJ \cos \beta /1000)^2$. The difference in elevation from A to B is then $AJ + JK - KL - HB$. The values of AJ and HB are recorded at the time of measurement. The correct difference in elevation between the two ends of the line is then the mean of the two values computed both ways either with or without taking into account curvature and refraction.

The accuracy of the determination of difference in elevation over a long distance is basically a function of the uncertainty of the atmospheric refraction and of the accuracy of the vertical angles. The slope distance obtained with EDM will be so accurate that no appreciable error will be introduced from this source. The effect of uncertainty in atmospheric refraction is held to a minimum by reciprocal observations. Bearing in mind that a second is a foot in 40 miles, then in a distance of say 2 miles the error of difference in elevation can be held to 0.20 ft if the vertical-angle accuracy is about 4″, which is obtainable with the precise theodolites discussed in Chapter 4. This error is reduced in proportion to a reduction in distance. If reciprocal observations are taken every 1000 m and the accuracy of vertical-angle measurements is of the order of 3″ to 4″, the error can be held to $1\frac{1}{2}$ to 2 cm or about 0.05 to 0.07 ft.

When applying trigonometric leveling to very long lines, the slope distance is measured using the intermediate or long-range EDMs, and reciprocal vertical angles are measured using precise theodolites. If possible, the reciprocal angles should be made simultaneously to eliminate the refraction uncertainty. If this proves unfeasible, then more than one set should be observed at different times to average out the errors due to this uncertainty.

EXAMPLE 3-1 The slope distance between two mountain peaks determined by EDM measurement is 76,963.54 ft. The vertical angle at the lower of the two peaks to the upper peak is +3° 02′ 05″. The reciprocal vertical angle at the upper peak is − 3° 12′ 55″. The height of the instrument and the height of the target at the ends of the line are assumed to be equal. Compute the difference in elevation between the two peaks first by using only the single vertical angles and applying correction for curvature and refraction, and second by using the average obtained by both vertical angles.

Solution: The difference in elevation using only the single vertical angle α is

$$\Delta H = 76,963.54 \sin 3° \, 02' \, 05'' + 0.0206\left(\frac{76,963.54}{1000} \cos 3° \, 02' \, 05''\right)^2$$

$$76,963.54 \sin 3° \, 02' \, 05'' = 76,963.54 \times 0.05294113 = 4074.54$$

$$0.0206\left(\frac{76,963.54}{1000} \cos 3° \, 02' \, 05''\right)^2 = 0.0206 \times 5907 = +121.68$$

$$\Delta H = +4196.22 \text{ ft}$$

The difference in elevation using only the single vertical angle β is

$$\Delta H = 76,963.54 \sin 3° \, 12' \, 55'' - 0.0206[(76,963.54/1000) \cos 3° \, 12' \, 55'']^2$$
$$= +4316.71 - 121.64$$
$$= 4195.07 \text{ ft.}$$

The average from the two solutions is +4195.64 ft.

The difference in elevation without taking into account the correction for curvature and refraction, and using the average, is

$$76,963.54 \sin 3° \, 02' \, 05'' = 4074.54$$
$$76,963.54 \sin 3° \, 12' \, 55'' = 4316.71$$

$$\Delta H = \text{average} = +4195.63 \text{ ft}$$

3-4 Direct Differential Leveling

The purpose of differential leveling is to determine the difference of elevation between two points on the earth's surface. The most accurate method of determining differences of elevation is with the spirit level and a rod, as shown in Fig. 3-4. It is assumed that the elevation of point A is 976 ft and that it is desired to determine the elevation of point B. The level is set up, as described in Section 3-23, at some convenient point so that the instrument is higher than both A and B. A leveling rod is held vertically at point A, which may be on the top of a stake or on some solid object, and the telescope is directed toward the rod. The vertical distance from A to a horizontal plane can be read on the rod where the horizontal cross hair of the telescope appears to coincide. If the rod reading is 7.0 ft, the plane of the telescope is 7.0 ft above point A. The elevation of this horizontal plane is 976 + 7 = 983 ft. The leveling rod is next held vertically at B, and the telescope is directed toward

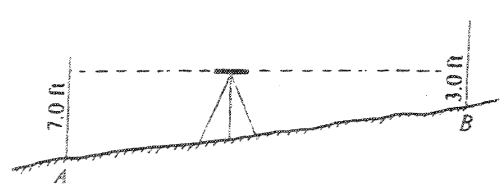


FIGURE 3-4 Direct differential leveling

the rod. The vertical distance from B to the same horizontal plane is given by the rod reading with which the horizontal cross hair appears to coincide. If the rod reading at B is 3.0 ft, point B is 3.0 ft below this plane and the elevation of B is 983 − 3 = 980 ft. The elevation of the ground at the point at which the level is set up need not be considered.

The same result may be obtained by noting that the difference in elevation between A and B is 7 − 3 = 4 ft, and that B is higher than A. The elevation of B equals the elevation of A plus the difference of elevation between A and B, or 976 + 4 = 980 ft.

3-5 Stadia Leveling

Stadia leveling combines features of trigonometric leveling with those of direct differential leveling. In stadia leveling, vertical angles are read by using the transit or theodolite, and horizontal distances are determined at the same time by means of the stadia hairs mentioned in Section 2-4. As in direct differential leveling, the elevation of the ground at the point at which the instrument is located is of no concern in the process. Stadia leveling is a rapid means of leveling when moderate precision is sufficient. It is described in detail in Section 14-10.

3-6 Leveling with Aneroid Barometer

The fact that atmospheric pressure, and hence the reading of a barometer, decreases as the altitude increases is utilized in determining differences of elevation. Because of transportation difficulties, the mercurial barometer is not used for survey purposes. Instead, the aneroid barometer or altimeter is used in surveys in which errors of 5 to 10 ft are of no consequence. Altimeters vary in size from that of an ordinary watch to one that is 10 or 12 in. in diameter. The dials sometimes have two sets of graduations, namely, feet or meters of elevation and inches or millimeters of mercury. The smaller altimeters can be read by estimation to about 10 ft. A larger type, one of which is shown in Fig. 3-5, is much more sensitive, and differences of elevation of 2 or 3 ft can be detected.

An altimeter is, of course, subject to natural changes in atmospheric pressure due to weather, and it is also subject to effects of temperature and humidity. For this reason altimeters should be used in groups of three or more.

In Fig. 3-6 an altimeter is maintained at L, which is a point of known elevation and is designated the low base. A second altimeter, whose reading has been compared with the one kept at the low base, is taken to H, which is a second point of known elevation and is designated the high base. At regular intervals, say every 5 min, the altimeters at the low and high bases are read. A third altimeter, which is referred to as the field altimeter or roving altimeter, is initially compared to the low-base altimeter and then taken to various points whose elevations are to be established. At each point the reading of the roving altimeter and the time are recorded. The elevation of any point can be determined from the reading of the roving altimeter at the point and the readings of the altimeters at the high and low

TABLE 3-1 Two-Base Altimeter Readings.
Indexing on Low Base; Low-Base Altimeter 1180; High-Base Altimeter 1189; Roving Altimeter 1188

Time	Low-Base Reading (ft)	Low-Base Elevation (ft)	High-Base Reading (ft)	High-Base Elevation (ft)	Roving-Altimeter Reading (ft)
1:55 P.M.	1180	225	1637	658	1360
2:15 P.M.	1186	225	1650	658	1425
2:20 P.M.	1184	225	1646	658	1452

Before either the high-base readings or the roving-altimeter readings are used for computing elevations, they must be corrected for index error. Thus the high-base readings must be reduced by 9 ft and the roving-altimeter readings by 8 ft. It is possible to make all three altimeters read the same at the time of indexing by physically adjusting the pointers, but this practice is not recommended. The corrected readings are shown here for convenience.

Time	Low-Base Reading (ft)	Corrected High-Base Reading (ft)	Corrected Roving-Altimeter Reading (ft)	Difference High-Low (ft)	Difference Roving-Low (ft)	Δh (ft)
1:55 P.M.	1180	1628	1352	448	172	166
2:15 P.M.	1186	1641	1417	455	231	220
2:20 P.M.	1184	1637	1444	453	260	249

The known difference in elevation between the high base and the low base is 658 − 225 = 433 ft. This value is represented by the length of the vertical line LH in Fig. 3-6. At the time of the first field reading, the difference between the corrected readings at the high and low bases is 448 ft. This value is represented by the length of the sloped line $L'H$. At the same time, the difference between the corrected readings at the field point and the low base is 172 ft, which is represented by the distance $L'P'$. Then, by proportion, the difference in elevation Δh between the low base and the field point, or the distance LP, is (172)(433)/448 = 166 ft. Similarly, the differences in elevation between the low base and the other two points are, respectively, (231)(433)/455 = 220 ft and (260)(433)/453 = 249 ft.

3-7 Types of Spirit Levels

The instrument most extensively used in leveling is the engineer's level. It consists essentially of a telescope to which a very accurate spirit level is attached longitudinally. The telescope is supported at the ends of a straight bar that is firmly secured at the center to the perpendicular axis on which it revolves. The level is supported on a tripod.

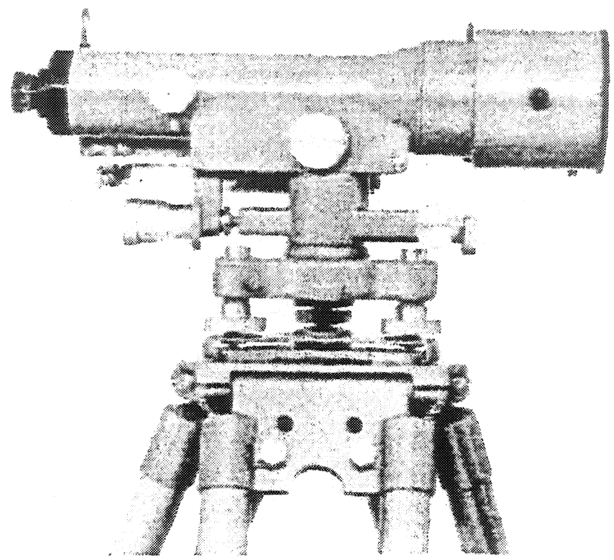


FIGURE 3-20 Precise tilting level with optical micrometer used for geodetic leveling. Courtesy of Leica, Inc.

3-16 Hand Level and Clinometer

The hand level, shown in Fig. 3-23, is a brass tube with a small level tube mounted on the top. A 45° mirror on the inside of the main tube enables the user to tell when it is being held horizontally. As the rod viewed through the level is not magnified. the length of sight is limited by the visibility of rod readings with the naked eye.

The hand level is used on reconnaissance surveys where extreme accuracy is unnecessary and in taping to determine when the tape is being held horizontally. It is also used to advantage for estimating how high or how low the engineer's level must be set to be able to read the leveling rod.

The clinometer, shown in Fig. 3-24, can be used in the same manner as the hand level. In addition it can be employed for measuring vertical angles where approximate results are sufficient.

3-17 Leveling Rods

In addition to the bar code rod used with the digital level, there are two other general classes of leveling rods: self-reading and target rods. A self-reading rod has painted graduations that can be read directly from the level. When a target rod is used, the target is set by the rodman as directed by the levelman, and the reading is then made by the rodman. Some types of rods can be used either as self-reading rods or as target rods.

The reading of a high rod is the distance from the base of the rod to the target. Thus, for the position shown in Fig. 3-25(b), the rod reading represents the distance h. When the target is set at 7 ft on the extension part of the rod while the rod is closed, the reading of the high rod is 7 ft, and the 7-ft graduation on the back of the rod is opposite the zero of the scale on the sleeve. As the rod is raised, the 7-ft mark moves upward, whereas the zero mark of the scale remains stationary since it is attached to the lower portion of the rod. The distance h' between these two marks therefore increases as the rod is extended further. Consequently, the distance h' is equal to the amount by which the target is raised above 7 ft, and for any high-rod reading the distance h is equal to 7 ft plus the distance h'. To obviate actual addition, the foot graduations on the back of the rod are numbered downward from 7 to 13. Thus, when the rod is extended 1 ft, the reading on the back is $7 + 1 = 8$ ft, and so on. Therefore the numbers must increase downward so the rod readings can become greater as the target is raised.

If the rod has been damaged by allowing the upper portion to slide down with a bang, it is possible that the reading on the back of the rod will be less than 7 ft when the rear section is in the lowered position. In this case the target should be set at the corresponding reading on the face of the rod before extending the rod.

3-19 Precise Leveling Rods

A precise rod is graduated on an invar strip that is independent of the main body of the rod except at the shoe at the bottom of the rod. The graduations are in yards, in feet, or in meters, and the smallest graduations are 0.01 yd, 0.01 ft, and 1 cm, respectively. A yard rod or a meter rod is also graduated in feet on the back of the rod. These foot graduations are painted directly on the main body of the rod. They serve as a check on the more precise readings taken on the invar strip, and help to prevent mistakes in rod readings. The precise rod is equipped with either a bull's-eye level or a pair of level vials at right angles to each other to show when the rod is vertical, and also with a thermometer. Front and back views of a precise rod are shown in Fig. 3-26.

3-20 Reading the Rod Directly

If the target is not used, the reading of the rod is made directly from the telescope. The number of feet is given by the red figure just below the horizontal cross hair when the level has an erecting telescope, or just above the horizontal cross hair when it has an inverting telescope. The number of tenths is shown by the black figure just below or above the hair, the position depending on whether the telescope is erecting or inverting. If the reading is required to the nearest hundredth, the number of hundredths is found by counting the divisions between the last tenth and the graduation mark nearest to the hair. If thousandths of a foot

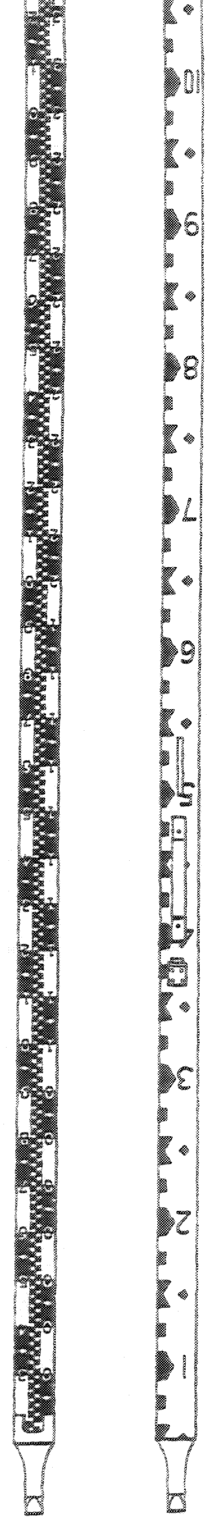


FIGURE 3-26 Precise leveling rod.

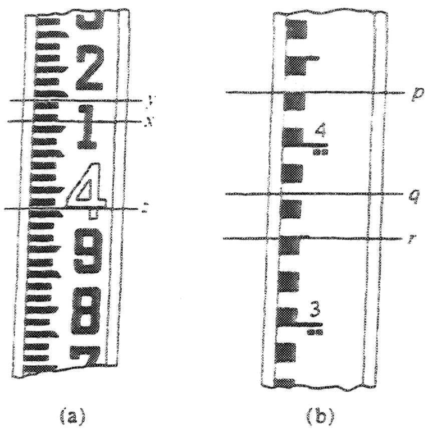


FIGURE 3-27 *Direct reading of rod.*
(a) Rod graduated to 0.01 ft.
(b) Rod graduated to 1 cm.

are required, the number of hundredths is equal to the number of divisions between the last tenth and the graduation mark on the same side of the hair as that tenth, and the number of thousandths is obtained by estimation. If the instrument is equipped with stadia hairs, care must be taken to read the correct (*middle*) horizontal cross hair.

The readings on the rod for the positions x, y, and z in Fig. 3-27(a) are determined as follows: For x, the number of feet below the cross hair is 4, the number of tenths below is 1, and the cross hair coincides with the first graduation above the tenth mark; consequently, the reading is 4.11 ft to the nearest hundredth, or 4.110 ft to the nearest thousandth. For y, the feet and tenths are again 4 and 1, respectively, and the hair is just midway between the graduations indicating 4 and 5 hundredths, therefore the reading to the nearest hundredth can be taken as either 4.14 or 4.15 ft. In determining the hundredths it is convenient to observe that the hair is just below the acute-angle graduation denoting the fifth hundredth, and it is therefore unnecessary to count up from the tenth graduation. If thousandths are required, the number of hundredths is the lower one, or 4; and since the hair is midway between two graduation marks on the rod and the distance between the graduations is 1 hundredth or 10 thousandths of a foot, the number of thousandths in the required reading is $\frac{1}{2} \times 10$, or 5. Hence the reading to the nearest thousandth is 4.145 ft. For z, the reading to the nearest hundredth is 3.96 ft and that to the nearest thousandth is 3.963 ft.

Direct high-rod readings are made with the rod fully extended, as the graduations on the face of the rod then appear continuous.

The metric rod shown in Fig. 3-27(b) is numbered every decimeter and graduated in centimeters. The double dot shown below the decimeter numbers indicates the readings are in the 2-m range. The readings for positions p, q, and r are, respectively, 2.430 m, 2.373 m, and 2.349 m.

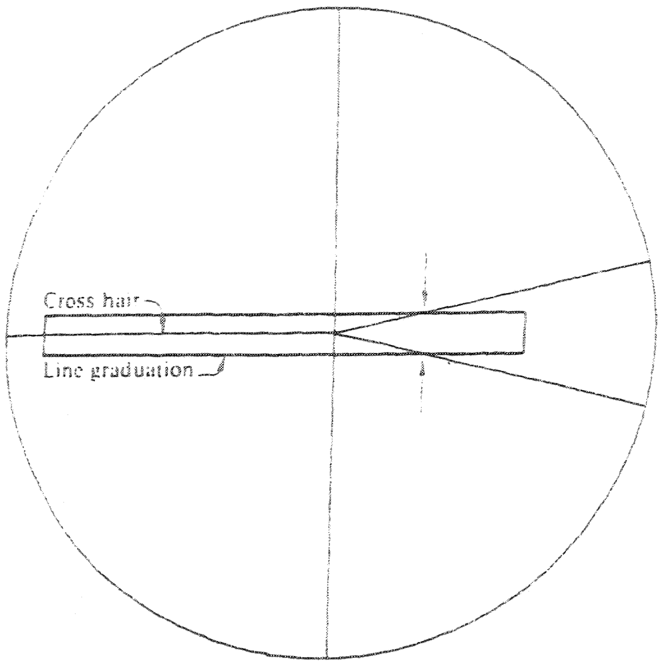


FIGURE 3-33 *Wedge type reticle. The thickness of the line graduation is greatly exaggerated. Symmetry is observed at the arrows.*

what more satisfactory design for use with the optical micrometer is the line graduation shown in Fig. 3-33. The level used with this type of design contains a horizontal cross hair that splits into a wedge halfway across the field of view. This wedge can be centered on the line graduation with a high degree of accuracy by the principle of symmetry.

3-23 Setting Up the Level

The purpose of direct leveling, as explained in Section 3-4, is to determine the difference of elevation between two points by reading a rod held on the points. These rod readings can be made by the levelman without setting the target, or the target can be set as directed by the levelman and the actual reading made by the rodman. At the instant the readings are made, it is necessary that the line of sight determined by the intersection of the cross hairs and the optical center of the objective be horizontal. In a properly adjusted instrument this line will be horizontal only when the bubble is at the center of the bubble tube.

The first step in setting up the level is to spread the tripod legs so that the tripod head will be approximately horizontal. The legs should be far enough apart to prevent the instrument from being blown over by a gust of wind, and they should be pushed into the ground far enough to make the level stable. Repairs to a damaged instrument are always expensive. For this reason, no instrument should be

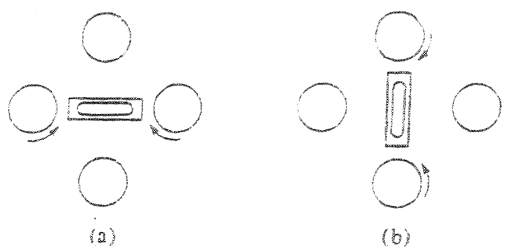


FIGURE 3-34 Manipulation of four leveling screws.

set up on a pavement or a sidewalk. If such a setup cannot be avoided, additional care should be exercised to protect it from possible mishaps.

If the level contains four leveling screws, the telescope is turned over either pair of opposite leveling screws, as shown in Fig. 3-34(a). The bubble is then brought approximately to the center of the tube by turning the screws in *opposite* directions. The level bubble moves in the direction of the left thumb, a point well worth remembering. No great care should be taken to bring the bubble exactly to the center the first time.

The next step is to turn the telescope over the other pair of screws and to bring the bubble exactly to the center of the tube by means of these screws. This is shown in Fig. 3-34(b). The telescope is now turned over the first pair of screws once more, and this time the bubble is centered accurately. The telescope is then turned over the second pair of screws, and if the bubble has moved away from the center of the tube, it is brought back to the center. When the instrument is finally leveled up, the bubble should be in the center of the tube when the telescope is turned over either pair of screws. If the instrument is in adjustment, the bubble should remain in the center as the telescope is turned in any direction.

The beginner will need considerable practice in leveling up the instrument. It is by practice alone that he is able to tell how much to turn the screws to bring the bubble to the center. The more sensitive the bubble, the more skill is required to center it exactly. For the final centering, when the bubble is to be moved only a part of a division, only one screw need be turned. The screw that has to be tightened should be turned if both are a little loose, and the one that has to be loosened should be turned when they are tight. When the telescope is finally leveled up, all four screws should be bearing firmly but should not be so tight as to put a strain in the leveling head. If the head of the tripod is badly out of horizontal, it may be found that the leveling screws turn very hard. The cause is the binding of the ball-and-socket joint at the bottom of the spindle. The tension may be relieved by loosening both screws of the other pair.

When a three-screw instrument is to be leveled, the level bubble is brought parallel with a line joining any two screws, such as *a* and *b* in Fig. 3-35(a). By rotating these two screws in opposite directions, the instrument is tilted about the axis *l*—*l*, and the bubble can be brought to the center. The level bubble is now brought perpendicular to the line joining these first two screws, as shown in Fig. 3-35(b). Then only the third screw, *c*, is rotated to bring the bubble to the center.

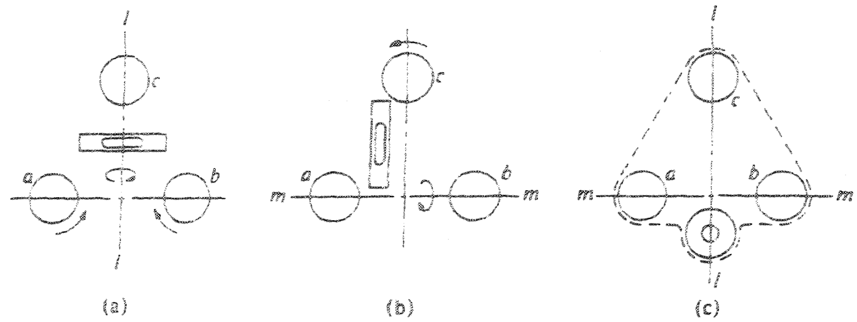


FIGURE 3-35 Manipulation of three leveling screws

This operation tilts the instrument about the axis m—m. The procedure is then repeated to bring the bubble exactly to the center in both directions.

When leveling a three-screw tilting level or an automatic level equipped with a bull's-eye bubble, as shown in Fig. 3-35(c), opposite rotations of screws a and b cause the bubble to move in the direction of the axis m—m. Rotation of screw c only causes the bubble to move in the direction of the axis l—l.

In walking about the instrument, the levelman must be careful not to step near the tripod legs, particularly when the ground is soft. Neither should any part of the level be touched as the readings are being made, because the bubble can be pulled off several divisions by resting the hand on the telescope or on a tripod leg. The bubble will not remain in the center of the tube for any appreciable length of time. The levelman should form the habit of always checking the centering of the bubble just before and just after making a reading. Only in this way can he be sure that the telescope was actually horizontal when the reading was made.

3-24 Signals

In running a line of levels, the levelman and the rodman must be in almost constant communication with each other. As a means of communication, certain convenient signals are employed. It is important that the levelman and the rodman understand these in order to avoid mistakes. When the target is used, it is set by the rodman according to signals given by the levelman. Raising the hand above the shoulder, so that the palm is visible, is the signal for raising the target; lowering the hand below the waist is the signal for lowering the target. The levelman, viewing the rod and the rodman through the telescope, should remember that he can see them much more distinctly than he can be seen by the rodman. Hence his signals should be such that there is no possible chance of misunderstanding. A circle described by the hand is the signal for clamping the target, and a wave of both hands indicates that the target is properly set, or all right. The signal for plumbing the rod is to raise one arm above the head and then to lean the body in the direction in which the rod should be moved.

3-25 Running a Line of Levels

In the preliminary example of direct leveling given in Section 3-4, it was assumed that the difference of elevation between the two points considered could be obtained by a single setting of the level. This will be the case only when the difference in elevation is small and when the points are relatively close together. In Fig. 3-36, rods at points A and K cannot be seen from the same position of the level. If it is required to find the elevation of point K from that of A, it will be necessary to set up the level several times and to establish intermediate points such as C, E, and G. These are the conditions commonly encountered in the field and may serve as an illustration of the general methods of direct leveling.

Let the elevation of the benchmark (abbreviated BM) at A be assumed as 820.00 ft. This is recorded as shown in the leveling notes of Fig. 3-37. The level is set up at B, near the line between A and K, so that a rod held on the BM will be visible through the telescope; the reading on the rod is found to be 8.42 ft. This reading is called a *backsight* reading, or simply a backsight (abbreviated BS), and is recorded as such in the notes. A backsight is the rod reading taken on a point of known elevation to determine the *height of instrument* (abbreviated HI). If the BS of 8.42 ft is added to the elevation of A, the HI is obtained. Thus HI = 820.00 + 8.42 = 828.42 ft. This is shown in the notes.

After the HI has been established, a point C called a *turning point*, is selected that is slightly below the line of sight. This point should be some stable unambiguous object, so that the rod can be removed and put back in the same place as many times as may be necessary. For this purpose, a sharp-pointed solid rock or a well-defined projection on some permanent object is preferable. If no such object is available, a stake or a railroad spike can be driven firmly in the ground and the rod held on top of it. After the turning point at C, designated TP-1,

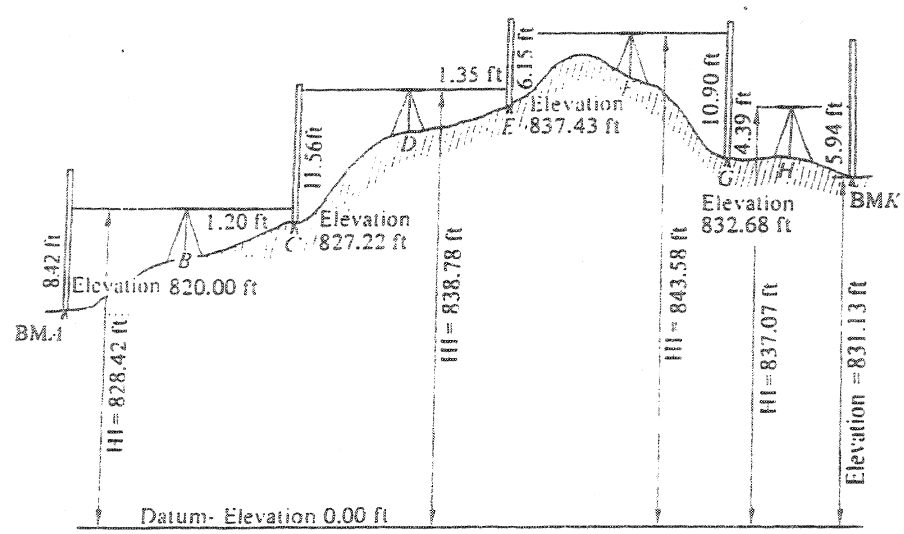
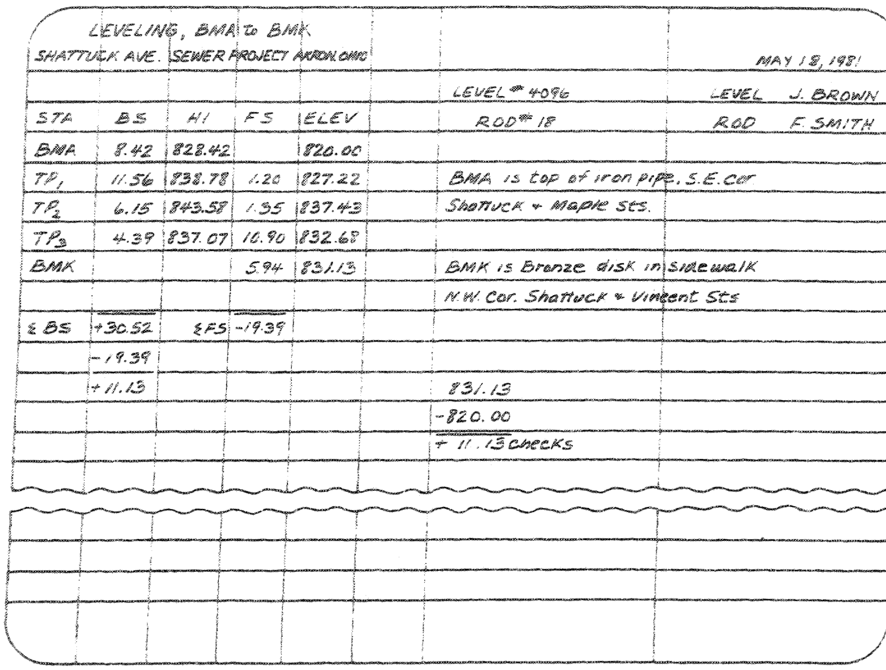


FIGURE 3-36 Direct leveling.



LEVELING, BMA to BMK
SHATTUCK AVE. SEWER PROJECT AKRON, OHIO

STA	BS	HI	FS	ELEV	LEVEL # 4096	MAY 18, 1981
					ROD # 18	LEVEL J. BROWN
						ROD F. SMITH
BMA	8.42	828.42		820.00		
TP_1	11.56	838.78	1.20	827.22	BMA is top of iron pipe, S.E. cor	
TP_2	6.15	843.58	1.35	837.43	Shattuck + Maple Sts.	
TP_3	4.39	837.07	10.90	832.68		
BMK			5.94	831.13	BMK is Bronze disk in sidewalk	
					N.W. Cor. Shattuck + Vincent Sts	
Σ BS	+30.52	Σ FS	−19.39			
	−19.39					
	+11.13				831.13	
					−820.00	
					+11.13 checks	

FIGURE 3-37 Level notes.

is set or selected, a reading is taken on the rod held on TP-1. If this reading is 1.20 ft, TP-1 is 1.20 ft below the line of sight, and the elevation of TP-1 is 828.42 − 1.20 = 827.22 ft, as shown in the notes. This rod reading is called a *foresight* (abbreviated FS). An FS is taken on a point of unknown elevation in order to determine its elevation from the height of instrument.

Occasionally, successive foresights and backsights are taken on an overhead point such as on a point in the roof of a tunnel. The foresight taken on such a point is *added* to the HI to obtain the elevation of the point. The backsight taken on the point is *subtracted* from the elevation of the point to determine the HI. Such readings must be carefully noted in the field notes.

While the rodman remains at *C*, the level is moved to *D* and set up as high as possible but not so high that the line of sight will be above the top of the rod when it is again held at *C*. This can be quickly checked by means of a hand level. The reading 11.56 ft is taken as a backsight. Hence, the HI at *D* is 827.22 + 11.56 = 838.78 ft. When this reading is taken, it is important that the rod be held on exactly the same point that was used for a foresight when the level was at *B*.

After the backsight on *C* has been taken, another turning point *E* is chosen, and a foresight of 1.35 ft is obtained. The elevation of *E* is 838.78 − 1.35 = 837.43 ft. The level is then moved to *F* and the backsight of 6.15 ft taken on *E*. The new HI is 837.43 + 6.15 = 843.58 ft. From this position of the level, a foresight of 10.90 ft is taken on *G*, the elevation of which is 843.58 − 10.90 = 832.68 ft. The level is then set up at *H*, from which position a backsight reading of 4.39 ft is taken on *G*, and a foresight reading of 5.94 ft is taken on the new BM at K. The final HI is 832.68 + 4.39 = 837.07 ft, and the elevation of K is 837.07 − 5.94 = 831.13 ft. As the starting elevation was 820.00 ft, the point K is 11.13 ft higher than A.

When using the digital level, the elevation of the beginning benchmark is keyed in. The microprocessor automatically adds backsight readings to obtain successive HIs, and subtracts foresights from preceding HIs to obtain the elevations of successive turning points and new benchmarks. These values, together with point designations keyed in by the instrumentman, are all stored in the memory module.

3-26 Checking Level Notes

To eliminate arithmetical mistakes in the calculation of HIs and elevations, the arithmetic should be checked on each page of notes. Adding backsights gives Σ BS; adding foresights gives Σ FS. Then Σ BS − Σ FS should equal the difference in elevation (DE) between the starting point to the last point on the page. This is shown in the notes of Fig. 3-37. Σ BS − Σ FS = +11.13 ft, and the calculated DE is also +11.13, which checks the arithmetic. The last point on the page should then be carried to the following page before the BS on that point is recorded in the notes.

3-27 Check Levels

Although the arithmetic in the reduction of the field notes may have been verified, there is no guarantee that the difference of elevation is correct. The difference of elevation is dependent on the accuracy of each rod reading and on the manner in which the field work has been done. If there has been any mistake in reading the rod or in recording a reading, the difference of elevation is incorrect.

The only way in which the difference of elevation can be checked is by carrying the line of levels from the last point back to the original benchmark or to another benchmark whose elevation is known. This is called "closing a level circuit." If the circuit closes on the original benchmark, the last point in the circuit, BMK in Fig. 3-36, must be used as a turning point; that is, after the foresight has been read on the rod at K from the instrument setup at H, the level must be moved and reset before the backsight is taken on K to continue the circuit to closure. Otherwise, if a mistake was made in reading the rod on the foresight to K from the setup at H, this mistake will not be discovered when checking the notes. A plan view of the level circuit between A and K in which the circuit is closed back on A is shown in Fig. 3-38(a). Note that the level has been reset between the FS taken on K and the BS taken on K. In Fig. 3-38(b) the level circuit has been continued to a known benchmark P to close the circuit. BMK is used as a turning point in this instance.

If the line of levels is carried back to BMA in the above example, on return the measured elevation of A should be 820.00 ft. The difference represents the error of closure of the circuit and should be very small. If a large discrepancy exists, the mistake may have been made in adding and subtracting backsight and foresight readings. This will be discovered on checking the notes. Otherwise there was a wrong reading of the rod or a wrong value was entered in the field notes. The adjustment of a level circuit based on the error of closure is discussed in Section 5-11.

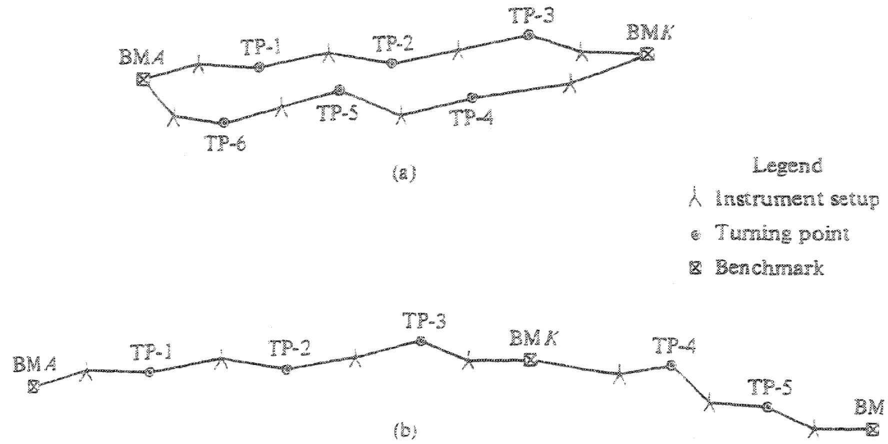


FIGURE 3-38 Closing on known benchmark.

3-28 Sources of Error in Leveling

The principal sources of error in leveling are instrumental defects, faulty mai lation of the level or rod, settling of the level or the rod, errors in sighting, mist in reading the rod or in recording or computing, errors due to natural sources personal errors.

3-29 Instrumental Errors

The most common instrumental error is caused by the level being out of ad ment. As has been previously stated, the line of sight of the telescope is horiz when the bubble is in the center of the tube, provided the instrument is in pe adjustment. When it is not in adjustment, the line of sight will either slope up or downward when the bubble is brought to the center of the tube. The va tests and adjustments of the level are given in Appendix B.

Instrumental errors can be eliminated or kept at a minimum by testing level frequently and adjusting it when necessary. Such errors can also be e nated by keeping the lengths of the sights for the backsight and foresight reac nearly equal at each setting of the level. Since it is never known just whe instrument goes out of adjustment, this latter method is the more certain should always be used for careful leveling.

In Fig. 3-39 the line of sight with the level at B should be in the horizontal $EBGK$. If the line of sight slopes upward as shown and a sight is taken on a r A, the reading is AF instead of AE. This reading is in error by the amount of I e_1. When the telescope is directed toward a rod held at C or D, the line of sight still slope upward through the same vertical angle if it is assumed that the bu remains in, or is brought to, the center of the tube. The rod reading taken on C be CH, which is in error by an amount $GH = e_2$. If the horizontal distances BE BG are equal, the errors e_1 and e_2 will be alike, and the difference between the

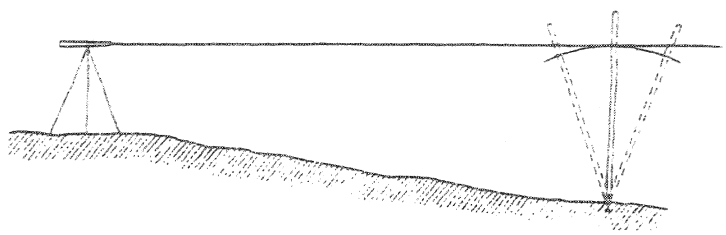


FIGURE 3-40 Waving the rod to obtain lowest reading.

Fig. 3-40. The least reading obtainable is the proper one. If the target is being used, the line dividing the colors should just coincide with the cross hair and then drop away from it. Errors from failure to hold the rod plumb will be much greater on readings near the top of the rod than for those near the bottom. For this reason more care should be exercised when making high-rod readings.

For careful work the lengths of the backsight and the foresight from the same setup should be kept nearly equal (see Section 3-29). If in ascending a steep hill the level is always kept on the straight line between the turning points, the distance to the backsight will be about twice as great as the distance to the foresight, and considerable error may result if the instrument is not in good adjustment. If there are no obstructions, these two distances can be kept nearly equal by setting the level some distance away from the straight line between the turning points. By thus zigzagging with the level, this source of error can be eliminated.

3-33 Mistakes in Reading the Rod, Recording, and Computing

A common mistake in reading the rod is to misread the number of feet or tenths or meters and decimeters. The careful levelman observes the foot and tenth marks both above and below the cross hair. On close sights, no foot mark may appear within the field of the telescope. In this case the reading can be checked by directing the rodman to place his finger on the rod at the cross hair, or if the reading is a high one, by having him slowly raise the rod until a foot mark appears in the telescope. In case of doubt the target can always be used.

Some instruments for precise leveling are equipped with three horizontal cross hairs. All three hairs are read at each sighting. If the hairs are evenly spaced, the difference between the readings of the upper and the middle hairs should equal the difference between the readings of the middle and lower hairs. This comparison is always made before the rodman leaves a turning point.

Where readings to thousandths of a foot are being made with the target, a common mistake in recording is to omit one or more ciphers from such readings as 5.004, and to record instead 5.04 or 5.4. Such mistakes can be avoided by making sure that there are three decimal places for each reading. Thus the second reading, if correct, should be recorded as 5.040, and the third as 5.400. If the values are not so recorded, the inference would be that the levelman was reading only to hundredths of a foot on the second reading and only to tenths on the third.

Other common mistakes of recording are the transposition of figures and the interchanging of backsight and foresight readings. If the levelman will keep the rodman at the point long enough to view the rod again after recording the reading, mistakes of the first type can often be detected. To prevent the interchange of readings, the beginner should remember that ordinarily the first reading taken from each position of the level is the backsight reading and that only one backsight is taken from any position of the level. Any other sights taken are foresights.

Mistakes in computations, as far as they affect the elevations of turning points and benchmarks, can be detected by checking the notes, as described in Section 3-26. This should be done as soon as the bottom of a page is reached, so that incorrect elevations will not be carried forward to a new page.

3-34 Errors Due to Natural Sources

One error due to natural sources is that caused by curvature and refraction, as described in Section 3-2. The error from this source amounts to but 0.0002 ft in a 100-ft sight (0.01 mm/30 m) and to about 0.002 ft in a 300-ft sight (0.7 mm/100 m). So for ordinary leveling it is a negligible quantity. It can be practically eliminated by keeping the backsight and foresight distances from the same setup equal. In precise leveling, if the backsight and foresight distances are not substantially equal, a correction is applied to the computed difference of elevation.

The familiar heat waves seen on a hot day are evidence of refraction, and when they are seen, refraction may be a significant source of error in leveling. When the heat waves are particularly intense, it may be impossible to read the rod unless the sights are much shorter than those usually taken. Refraction of this type is much worse close to the ground. For careful work it may be necessary to discontinue the leveling for 2 or 3 hr during the middle of the day. It may also be possible to keep the error from this source at a low figure by taking shorter sights and by so choosing the turning points that the line of sight will be at least 3 or 4 ft or 1 m above the ground.

Better results will usually be obtained when it is possible to keep the level shaded. If the sun is shining on the instrument, it may cause an unequal expansion of the various parts of the instrument; or if it heats one end of the bubble tube more than the other, the bubble will be drawn to the warmer end of the tube. For precise work the level must be protected from the direct rays of the sun.

To guard against changes in the length of the leveling rod from variations in temperature, the graduations on rods used for precise leveling are placed on strips of invar, which has an extremely small coefficient of expansion. For ordinary leveling, errors from this source are negligible.

3-35 Personal Errors

Some levelmen consistently tend to read the rod too high or too low. This may be caused by defective vision or by an inability to decide when the bubble is properly centered. In general, personal errors tend to be compensating, although some individuals may manipulate the level in such a way as to cause them to be cumulative.

Such a tendency can be discovered by leveling over a line that has been previously checked by several different parties or by running a number of circuits of levels, each of which begins and ends on the same point.

3-36 Limits of Error

If care is used in leveling, most of the errors will tend to be random. For this reason the error in any line can be expected to be proportional to the number of setups. Since the number of setups per mile of levels will be nearly constant, the error will also be proportional to the distance in miles. The precision that is being attained can be determined by comparing the two differences of elevation obtained by running levels in both directions over a line.

Level lines are classified as first order, second order, third order, or fourth order in accordance with the field procedures used and also with the agreement between the results of leveling in both directions over a line. Table 3-2 outlines the standards for all but fourth-order accuracy in vertical control surveys and gives the principal uses to which these levels of accuracy are applied. Trigonometric and barometric leveling are considered as being of fourth-order accuracy or less.

3-37 Reciprocal Leveling

In leveling across a river or a deep valley, it is usually impossible to keep the lengths of the foresight and the backsight nearly equal. In such cases reciprocal leveling is used, except where approximate results are sufficient. The difference of elevation between points on the opposite sides of the river or valley is then obtained from two sets of observations.

The method is illustrated by Fig. 3-41. The level is first set up at L_1 and rod readings are taken on the two points A and B. From these readings a difference of elevation is obtained. The level is then taken across the stream and set at such a position L_2 that $L_2B = L_1A$ and $L_2A = L_1B$. From this second position, readings are again taken on A and B, and a second difference of elevation is obtained. It is probable that these two differences will not agree. Both may be incorrect because of instrumental errors and curvature and refraction. However, the true difference of elevation should be very close to the mean of the two differences thus obtained.

The accuracy of this method will be increased if two leveling rods can be used, so that no appreciable time will elapse between the backsight and the foresight readings. It will be further increased by taking a number of rod readings on the more distant point and using the average of these readings, rather than depending on a single observation. If the distance AB is very great, it is important that the atmospheric conditions be the same for both positions of the level. Otherwise a serious error may be introduced by a changed coefficient of refraction.

EXAMPLE 3-2 In Fig. 3-41, with the level set at L_1, a backsight of 5.13 ft is taken on the rod held at A and a foresight of 7.25 ft is taken on the rod held at B. With the level at L_2, a backsight of 4.98 ft is taken on A, and a foresight of 6.82 ft is taken on B. The elevation of A is 298.72 ft. Determine the elevation of B.

foresight intervals, 27.845 ft. The difference between Σ FS intervals and Σ BS intervals is $27.845 - 22.464 = 5.381$ ft. Therefore the observed value of Σ FS means must be corrected by an amount equal to the C factor times the difference between the intervals, that is, by $-0.0112 \times 5.381 = -0.060$ ft. The corrected value of Σ FS means is thus $12.556 - 0.060 = 12.496$ ft. The corrected difference in elevation is then Σ BS means $-$ Σ FS means $= 31.422 - 12.496 = +18.926$ ft.

3-40 Profile Levels

The purpose of profile leveling is to determine the elevations of the ground surface along some definite line. Before a railroad, highway, transmission line, sidewalk, canal, or sewer can be designed, a profile of the existing ground surface is necessary. The route along which the profile is run may be a single straight line, as in the case of a short sidewalk; a broken line, as in the case of a transmission line or sewer; or a series of straight lines connected by curves, as in the case of a railroad, highway, or canal. The data obtained in the field are usually employed in plotting the profile. This plotted profile is a graphical representation of the intersection of a vertical surface or a series of vertical surfaces with the surface of the earth, but it is generally drawn so that the vertical scale is much larger than the horizontal scale in order to accentuate the differences of elevation. This is called vertical exaggeration.

3-41 Stations

The line along which the profile is desired must be marked on the ground in some manner before the levels can be taken. The common practice is to set stakes at some regular interval—which may be 100, 50, or 25 ft, or 30, 20, or 10 m depending on the regularity of the ground surface at each of these points. The beginning point of the survey is designated as station 0. Points at multiples of 100 ft or 100 m from this point are termed *full stations*. Horizontal distances along the line are most conveniently reckoned by the station method. Thus points at distances of 100, 200, 300, and 1000 ft from the starting point of the survey are stations 1, 2, 3, and 10, respectively. Intermediate points are designated as *pluses*. A point that is 842.65 ft from the beginning point of the survey is station 8 + 42.65. If the plus sign is omitted, the resulting figure is the distance, in feet, from station 0.

In the remainder of this chapter, reference will be made to stations of 100 ft. However, 100-m stations are handled in exactly the same way if the leveling is performed in metric units.

When the stationing is carried continuously along a survey, the station of any point on the survey, at a known distance from any station or plus, can be calculated. Thus a point that is 227.94 ft beyond station 8 + 42.65 is 842.65 + 227.94 = 1070.59 ft from station 0 or at station 10 + 70.59 the distance between station 38 + 66.77 and station 54 + 43.89 is 5443.89 − 3866.77 = 1577.12 ft.

In the case of a route survey, the stationing is carried continuously along the line to be constructed. Thus, if the survey is for a highway or a railroad, the stationing will be carried around the curves and will not be continuous along the straight lines, which are eventually connected by curves. For the method of stationing that is used in surveys of this sort, see Chapter 13.

3-42 Field Routine of Profile Leveling

The principal difference between differential and profile leveling is in the number of foresights, or −S readings, taken from each setting of the level. In differential leveling only one such reading is taken, whereas in profile leveling any number can be taken. The theory is exactly the same for both types of leveling. A backsight, or +S reading, is taken on a benchmark or point of known elevation to determine the height of the instrument. The rod is then held successively on as many points, whose elevations are desired, as can be seen from that position of the level, and rod readings, called *intermediate foresights* (IFS), are taken. The elevations of these points are calculated by subtracting the corresponding rod readings from the height of the instrument (HI). When profile leveling with the digital level, the instrumentman must indicate via the keyboard that intermediate foresights are being observed from the current instrument setup in order that all IFSs are subtracted from the current HI. When no more stations can be seen, a foresight is taken on a turning point, the level is moved forward, and the process is repeated. Leveling rods up to 25 ft long are used in profile leveling to accommodate low spots along the line.

The method of profile leveling is illustrated in Fig. 3-48. The level having been set up, a sight is taken on a benchmark, not shown in the sketch. Intermediate foresights are then taken on stations 0, 1, 2, 2 + 65, 3, and 4. The sight is taken at station 2 + 65 because there is a decided change in the ground slope at that point. The distance to this point from station 2 is obtained either by pacing or by taping, the better method depending on the precision required. To determine the elevation of the bottom of the brook between stations 4 and 5, the level is moved forward after a foresight reading has been taken on the turning point just beyond station 4. With the level in the new position, a backsight is taken on the turning point, intermediate foresights are taken on stations 4 + 55, 4 + 63, 4 + 75, 5, 5 + 70, 6, 6 + 25, and 7, and lastly a foresight is taken on a turning point near station 7. From the third setup, a backsight is taken on the turning point, intermediate foresights are taken on stations 8, 8 + 75, 9, 10, 10 + 40, and 11, and a foresight is taken on a turning point near station 11. From the final setup shown in the figure, a backsight is taken on this turning point and intermediate foresights are taken on stations 12 and 13. Finally, a foresight is taken on a benchmark not shown in the sketch.

Readings have thus been taken at the regular 100-ft stations and at intermediate points wherever there is a decided change in the slope. The level has not necessarily been set on the line between the stations. In fact, it is usually an advantage to have the level from 30 to 50 ft away from the line, particularly when readings must be taken on intermediate points. More of the rod will then be visible through the telescope and the reading can be made more easily and quickly.

Benchmarks are usually established in the project area by differential leveling prior to running the profile leveling. When running the profile leveling, backsights and foresights on benchmarks and turning points must be taken with the same accuracy as that used to establish the elevations of the project benchmarks, usually to the hundredth of a foot (0.001 m). This is necessary to maintain the overall accuracy of the profile leveling, which minimizes the accumulation of errors. If intermediate foresights along profiles are taken along bare ground, they need only be read to the nearest tenth of a foot (1 cm). However, if the entire profile is a paved surface, it may be required to read the intermediate foresights to the hundredth of a foot, depending on the purpose of the profile. The profile leveling is then adjusted between previously established project benchmarks.

If benchmarks have not been established in advance, they should be established as the work progresses. Benchmarks may be from 10 to 20 stations apart when the differences of elevation are moderate, but the vertical intervals between benchmarks should be about 20 ft where the differences of elevation are considerable. These benchmarks should be so located that they will not be disturbed during any construction that may follow. Their elevations should be verified by running check levels.

The notes for recording the rod readings in profile leveling are the same as those for differential leveling except for the addition of a column for intermediate foresights. The notes and calculations for a portion of the profile leveling shown in Fig. 3-48 are given in Fig. 3-49.

3-43 Plotting the Profile

To facilitate the construction of profiles, paper prepared especially for the purpose is commonly used. This has horizontal and vertical lines in pale green, blue, or orange, so spaced as to represent certain distances to the horizontal and vertical scales. Such paper is called *profile paper*. If a single copy of the profile is sufficient, a heavy grade of paper is used. When reproductions are necessary, either a thin paper or tracing cloth is available. The common form of profile paper is divided into $\frac{1}{4}$-in. squares by fairly heavy lines. The space between each two such horizontal lines is divided into five equal parts by lighter horizontal lines, the distance between these light lines being $\frac{1}{20}$ in. To accentuate the differences of elevation, the space between two horizontal lines can be considered as equivalent to 0.1, 0.2, or 1.0 ft, and the space between two vertical lines as 25, 50, or 100 ft, according to the total difference of elevation, the amount of vertical exaggeration desired, the length of the line, and the requirements of the work.

To aid in estimating distances and elevations, each tenth vertical line and each fiftieth horizontal line are made extra heavy. Profile paper showing the profile for the level notes given in Fig. 3-49 is illustrated in Fig. 3-50. The elevation of some convenient extra-heavy horizontal line is assumed to be 900 ft, and a heavy vertical line is taken as station 0. Each division between horizontal lines represents 1 ft, and each division between vertical lines represents 100 ft, or one station. As the elevation or station of each printed line is known, the points on the ground surface can be plotted easily. When these points are connected with a smooth line, an accurate representation of that ground surface should result.

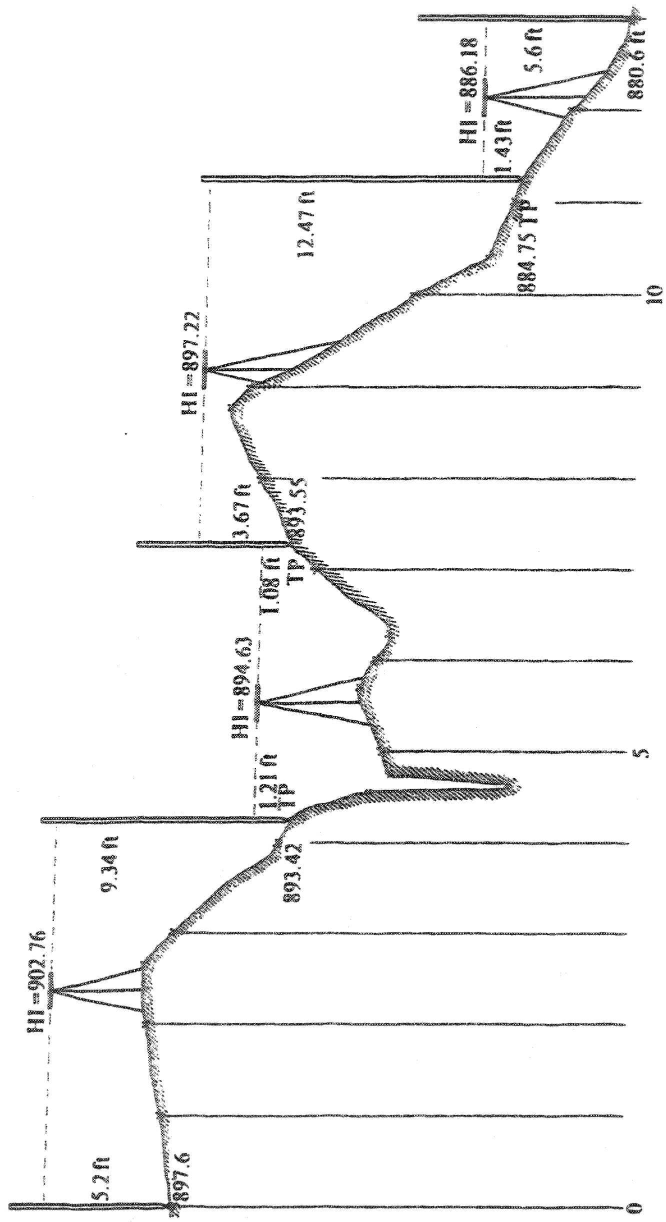


FIGURE 3-48 Profile leveling.

earthwork at a minimum, to have the grade line straight for considerable distances and its inclination within the allowable limit, and to have the excavation and the embankment balance over reasonably short stretches (see Section 17-17). To determine the best grade line may require considerable study, but the saving of even a few hundred cubic yards of grading will pay for many hours of such study.

Where changes in the inclination of the grade line occur, the straight grades are connected by vertical curves (see Section 13-15).

3-45 Rate of Grade

The inclination of the grade line to the horizontal can be expressed by the ratio of the rise or fall of the line to the corresponding horizontal distance. The amount by which the grade line rises or falls in a unit of horizontal distance is called the *rate of grade* or the *gradient*. The rate of grade is usually expressed as a percentage, that is, as the rise or fall in a horizontal distance of 100 ft. If the grade line rises 2 ft in 100 ft, it has an ascending grade of 2%, which is written +2%. If the grade line falls 1.83 ft in 100 ft, it has a descending grade of 1.83%, which is written −1.83%. The sign + indicates a rising grade line and the sign − indicates a falling grade line.

The rate of grade is written along the grade line on the profile. The elevation of grade is written at the extremities of the line and also at each point where the rate of grade changes. It is common practice to enclose in small circles the points on the profile where the rate of grade changes.

The rate of grade, in percent, is equal to the total rise or fall in any horizontal distance divided by the horizontal distance expressed in stations of 100 ft. The total rise or fall of a grade line in any given horizontal distance is equal to the rate of grade, in percent, multiplied by the horizontal distance in stations. The horizontal distance, in stations of 100 ft, in which a given grade line will rise or fall a certain number of feet, is equal to the amount of the required rise or fall divided by the rate of gradient in percent.

EXAMPLE 3-3 The grade elevation in a construction project at station 40 + 50 is to be 452.50 ft. The elevation along this same grade line at station 52 + 00 is to be 478.80 ft. What is the percent grade? What is the grade elevation at station 48 + 25? Refer to Fig. 3-52.

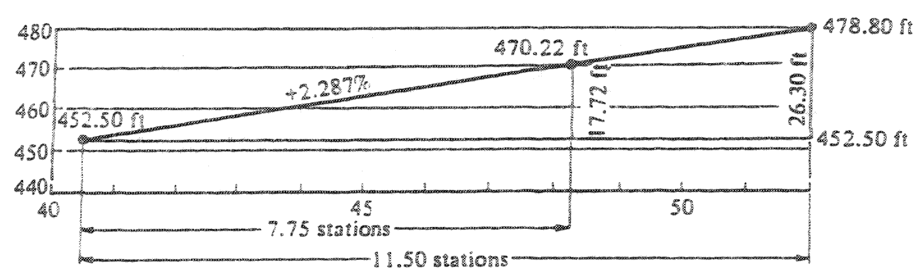


FIGURE 3-52 *Calculation of grade and elevation.*

Solution: The difference in elevation between the two points is 478.80 – 452.50 = +26.30 ft. The difference in stations is 52 – 40.50 = 11.50 stations. The percent grade is thus +26.30/11.50 = +2.287%. Station 48 + 25 lies 7.75 stations beyond station 40 + 50. The rise of the grade line between these two stations is 7.75 × 2.287 = +17.72 ft. The grade elevation at station 48 + 25 is thus 452.50 + 17.72 = 470.22 ft.

PROBLEMS

3-1. A backsight of 3.0455 is taken on a point 60 m from the level. A foresight of 1.1508 m is then taken on a point 220 m from the level. Compute the correct difference in elevation, taking into account the effect of curvature. Neglect the effect of refraction on the line of sight.

3-2. Sighting across a lake 13 miles wide through a pair of binoculars, what is the height of the shortest tree on the opposite shore whose tip the observer can see if his eyes are 5 ft 3 in. above the shore line on which he stands?

3-3. A backsight of 3.865 ft is taken on a point 20 ft from the level. A foresight of 2.680 ft is then taken on a point 220 ft from the level. Compute the correct difference in elevation between the two points, taking into account the effect of curvature. Neglect the effect of refraction.

3-4. A sailor is standing in the crow's nest of a ship. His eyes are 120 ft above the level of the water. Looking through his binoculars, he sees the tip of the mast of an approaching ship. The mast is 150 ft above the water. What is the distance, in miles, between the two ships?

3-5. A vertical angle of –2° 40′ 30″ is read on a target that is 11.0 ft above ground station *B*. The telescope of the instrument is 5.0 ft above ground at station *A*. *AB* = 482.5 m. The elevation of station *B* is 345.46 m. Compute the elevation of station *A* in meters.

3-6. A slope distance between two points *P* and *Q* is measured using a laser EDM, giving a corrected slope distance of 45,580.50 ft. The elevation of the lower point *P* is 1542.85 ft. Reciprocal vertical angles are measured at *P* = +2° 12′ 15″, and at *Q* = –2° 19′ 15″. Assume the instrument and target are at the same height above ground at the two stations. What is the elevation of point *Q*?

3-7. In problem 3-6, using only the vertical angle at *Q*, together with the normal effect of curvature and refraction, compute the elevation of *Q*.

3-8. A vertical angle of +12° 52′ 25″ is measured to a station at the top of a hill from an instrument set up 2250 feet away from the hilltop station, measured horizontally. The height of the target over the station is 12.28 ft. The telescope of the instrument is 5.20 ft above the lower station, whose elevation is 322.64 ft. Assuming normal refraction conditions, what is the elevation of the hilltop station?

3-9. The slope distance between two points *C* and *D* is measured as 12,476.82 m. A vertical angle of +3° 12′ 22″ is measured from *C* to *D*. The instrument is 1.63 m above ground at *C*; the target is 1.63 m above ground at *D*. Compute the difference in elevation from *C* to *D*.

3-10. Three altimeters *A*, *B*, and *C* read as follows when set on a benchmark whose elevation is 457 ft; $R_A = 1405$, $R_B = 1423$, $R_C = 1413$. Altimeter *A* is kept at the benchmark; altimeter *B* is taken to a benchmark at an elevation of 1848 ft; altimeter *C* is used as a field altimeter. The following readings were taken:

Time	Altimeter A	Altimeter B	Altimeter C	Point
2:35	1413	2797	1723	1
3:00	1411	2803	1947	2
3:05	1419	2811	1943	3
3:50	1413	2807	1568	4
4:05	1411	2801	1689	5

Determine the elevations of the five field points.

3-11. If the sensitiveness of a bubble on an engineer's level is 28" and the graduations on the tube are 2 mm apart, what is the radius of curvature of the bubble tube in meters?

3-12. A level is set up, and the end of the level bubble is carefully brought to a graduation on the tube by manipulating the leveling screws. A reading of 1.6685 m is taken on a rod held 110 m from the level. The level is then tilted so as to cause the end of the bubble to move over six divisions, and a second rod reading of 1.7466 m is taken. The graduations are 2 mm apart. What is the sensitiveness of the bubble, and what is the radius of curvature of the bubble tube in meters?

3-13. A level is set up 195 ft from a leveling rod. One end of the bubble is carefully brought to a graduation on the tube by means of the leveling screws. A reading of 4.891 ft is taken on the rod. The bubble is moved through five divisions, and a second rod reading is 4.742 ft. The spacing of the graduations is 2 mm. What is the sensitiveness of the bubble, and what is the radius of curvature of the bubble tube, in feet?

3-14. Complete the accompanying set of level notes recorded when running differential levels between two benchmarks. Check the notes to detect arithmetical mistakes.

Station	BS	HI	FS	Elevation (ft)
BM27	4.22			182.64
TP1	6.80		8.91	
TP2	3.16		0,08	
TP3	5.05		4.62	
TP4	12.95		3.16	
BM28	0.16		11.18	
TP5	2.28		7.74	
TP6	6.42		3.02	
TP7	10.98		5.92	
TP8	8.05		6.22	
TP9	8.15		5.80	
BM29	6.62		5.05	
TP10	4.85		5.54	
BM30			10.90	

3-15. The line of sight of a level falls at the rate of 0.122 ft/100 ft when the level bubble is centered. A backsight of 4.782 ft is taken on point *A*, which is 75 ft from the level. The elevation of *A* is 425.238 ft. A foresight of 5.162 ft is taken on point *B*, which is 190 ft from the level. If the bubble was centered for both the backsight and the foresight, what is the elevation of *B*? Neglect curvature and refraction.

3-16. Assume that an area is to be graded to a level surface, and that the grading is to be controlled by grade stakes. The line of sight of the level rises at the rate of 0.086 ft/100 ft when the bubble is centered. A backsight is taken on a point that defines the grade elevation, and the rod reads 4.28 ft. The backsight point is 32 ft from the level. A series of grade stakes are set at the following distances from the level:

Stake	Distance (ft)	Stake	Distance (ft)	Stake	Distance (ft)
1	20	6	114	11	15
2	68	7	137	12	70
3	36	8	190	13	190
4	102	9	166	14	230
5	128	10	202	15	144

Compute the correct rod readings that will put the foot of the rod at grade at each of these points.

3-17. Assume the graduations of a 13-ft Philadelphia rod to be in the plane of the front of the rod and that the shoe or foot of the rod is $1\frac{7}{8}$ in. wide front to back. What is the effect on a rod reading of 11.000 ft if the rod is held on a flat surface and is waved forward toward the instrument such that the top of the rod moves 10 in.? What is the effect if the rod is waved backward away from the level by the same amount (nearest 0.001 ft)?

3-18. Complete the accompanying set of level notes in running differential levels between two benchmarks. Check the notes for arithmetical mistakes.

Station	BS	HI	FS	Elevation (m)
BM101A	2.087			47.466
TP1	2.110		0.884	
TP2	1.846		1.462	
TP3	0.484		1.598	
TP4	0.256		1.778	
BM102A	1.411		2.324	
TP5	1.798		1.780	
TP6	2.024		1.752	
TP7	3.198		1.046	
TP8	3.148		2.353	
TP9	2.862		3.046	
BM103A	2.140		3.021	
TP10	0.898		2.106	
BM104A			0.989	

3-19. The instrumentman observes a low reading of 11.260 ft and a high reading of 11.285 ft when the rodman waves the rod. How far is the rod out of plumb at the 13-ft mark when the high reading is taken?

3-20. Complete and check the accompanying set of profile level notes.

Station	BS	HI	FS	IFS	Elevation (ft)
BM38	6.94				491.29
22 + 00				6.4	
23 + 00				6.0	
23 + 71				3.2	
24 + 00				2.4	
24 + 18				2.0	
TP1	0.92		8.72		
25 + 00				4.6	
26 + 00				12.9	
26 + 80				9.8	
27 + 00				9.5	
27 + 78				12.4	
28 + 00				12.4	
TP2	5.63		10.10		
28 + 27				3.3	
28 + 47				0.9	
29 + 00				2.8	
30 + 00				6.1	
30 + 60				7.3	
31 + 00				8.6	
32 + 00				10.2	
BM39			9.83		

3-21. Using a horizontal scale of 1″ = one station and a vertical scale of 1″ = 10 ft, plot the profile of the line from the notes of Problem 3-20.

3-22. The grade elevation at station 25 + 00.00 is to be 946.52 ft. What will be the grade elevation at station 34 + 86.20 if the grade line rises at the rate of 1.785% between these two points?

3-23. The grade elevation of station 153 + 50 is 763.60 ft, and the grade elevation of station 161 + 75 is 785.44 ft. What is the grade, in percent, of a slope joining these two stations (nearest 0.001%)?

3-24. Compute the slope of the line of sight of a level from the following observations taken in each instance with the level bubble centered. From the first instrument setup, the reading on a rod at A is 3.729 ft and that on a rod at B is 4.286. The distance from the level to A is 25 ft, and the distance to B is 200 ft. From the second instrument setup, the rod reading at A is 4.368 ft and that at B is 4.908 ft. The distance to A is 185 ft, and that to B is 15 ft.

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7 Direction of a Line

7-1 Astronomical Meridian

A plane passing through a point on the surface of the earth and containing the earth's axis of rotation defines the *astronomical meridian* at the point. The direction of this plane may be established by observing the position of the sun or a star, as described in Chapter 12. By popular usage the intersection of this meridian plane with the surface of the earth is known as the *true meridian*.

7-2 Magnetic Meridian

The earth acts very much like a bar magnet with a north magnetic pole located considerably south of the north pole defined by the earth's rotational axis. The magnetic pole is not fixed in position but changes its position continually. A magnetized needle freely suspended on a pivot will come to rest in a position parallel to the magnetic lines of force acting in the vicinity of the needle. Generally, the greatest component of the magnetic force at a point is that created by the earth's magnetic field, but other components may be created by other magnetic fields such as those around electric power lines, reinforcing bars in roads and structures, and iron deposits. The direction of the magnetized needle defines the *magnetic meridian* at the point at a specific time. Unlike the true meridian, whose direction is fixed, the magnetic meridian varies in direction.

A gradual shift in the earth's magnetic poles back and forth over a great many years causes a secular change, or variation, which amounts to several degrees in a cycle. An annual variation of negligible magnitude is experienced by the earth's magnetic field. A daily variation causes the needle to swing back and forth through an angle of not much more than one-tenth of a degree each day.

Local attraction, the term applied to the magnetic attractions other than that of the earth's magnetic field, may change at a given location. This change may be caused by a variation in the voltage carried by a power line or by a gradual increase or decrease of a magnetic field in reinforcing bars, wire fences, underground utility pipes, or other metal parts.

7-3 Assumed Meridian

For convenience in a survey of limited extent, any line of the survey may be assumed to be a meridian or a line of reference. An assumed meridian is usually taken to be in the general direction of the true meridian.

7-4 Convergence of Meridians

True meridians on the surface of the earth are lines of geographic longitude, and they converge toward each other as the distance from the equator toward either of the poles increases. The amount of convergence between two meridians in a given vicinity depends on (1) its distance north or south of the equator and (2) the difference between the longitudes of the two meridians. Magnetic meridians tend to converge at the magnetic poles, but the convergence is not regular and is not readily obtainable.

7-5 Grid Meridian

Another assumption is convenient in a survey of limited extent: When a line through one point of the survey has been adopted as a reference meridian, whether true or assumed north, all the other meridians in the area are considered to be parallel to the reference meridian. This assumption eliminates the necessity for determining convergence. The methods of plane surveying assume that all measurements are projected to a horizontal plane and that all meridians are parallel straight lines. These are known as *grid meridians*.

Two basic systems of grids are used in the United States to allow plane surveying to be carried statewide without any appreciable loss of accuracy. In each separate grid one true meridian is selected. This is called the *central meridian*. All other north-south lines in that grid are parallel to this line. The two systems, which are known as the *Lambert conformal projection* and the *transverse Mercator projection*, are the subject of Chapter 11.

7-6 Azimuth of a Line

The azimuth of a line on the ground is the horizontal angle measured from the plane of the meridian to the vertical plane containing the line. Azimuth gives the direction of the line with respect to the meridian. It is usually measured in a clockwise direction with respect to either the north meridian or the south meridian. In astronomical and geodetic work, azimuths are measured from the south meridian. In plane surveying, azimuths are generally measured from north.

A line may have an azimuth between 0° and 360°. In Fig. 7-1 the line *NS* represents the meridian passing through point *O*, with *N* toward the north. The azimuth

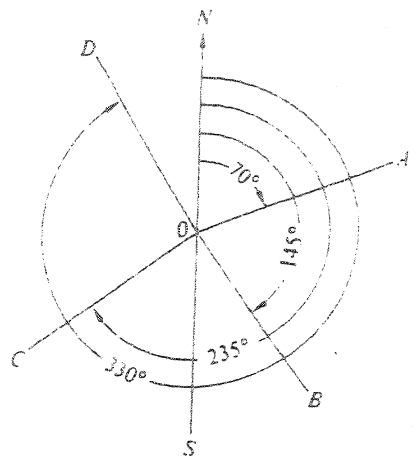


FIGURE 7-1 Azimuths.

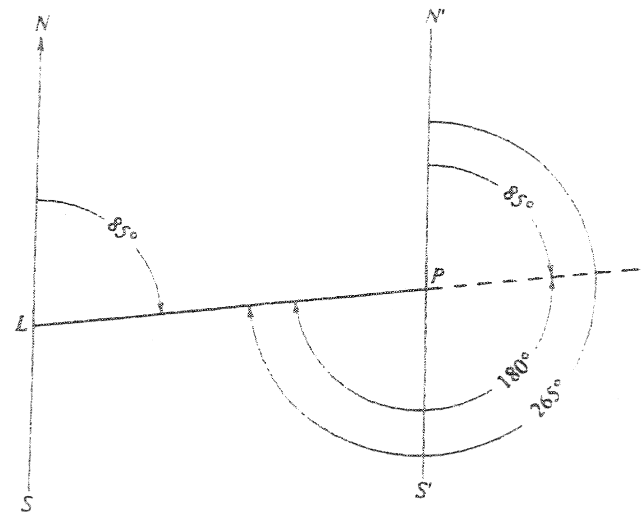


FIGURE 7-2 Relationship between azimuth and back azimuth.

of the line *OA* measured from the north is 70°; that of *OB* is 145°; that of *OC* is 235°; and that of *OD* is 330°. Azimuths are called *true* azimuths when measured from the true meridian, *magnetic* azimuths when measured from the magnetic meridian, *assumed* azimuths when referred to an arbitrary north-south line, and *grid* azimuths when referred to the central meridian in a grid system.

In the field of geodesy and geodetic surveying, the azimuth of a line is termed *geodetic* azimuth. It differs slightly from the true azimuth because a line at a point normal to the reference spheroid (see Sections 1-2 and 9-9) does not exactly follow the direction of gravity at that point, whereas true azimuth is related to the local vertical line.

7-7 Back Azimuth

When the azimuth of a line is stated, it is understood to be that of the line directed from an original point to a terminal point. Thus a line *LP* has its origin at *L* and its terminus at *P*. If the azimuth of *LP* is stated as 85°, then the azimuth of the line *PL* must have some other value since *LP* and *PL* do not have the same direction. One direction is the reverse of the other. For the purpose of discussing back azimuth, consider the line *LP* in Fig. 7-2. In the diagram *NS* is the meridian through *L*, and *N'S'* is the meridian through *P*. According to the assumption in plane surveying, the two meridians are parallel to each other. If the azimuth of *LP* is 85°, then the azimuth of *PL* is 85° + 180°, or 265°. Thus the back azimuth of *LP* is the same as the azimuth of *PL*. In Fig. 7-3 the azimuth of *CD* is 320° and the back azimuth of *CD*, which is the azimuth of *DC*, is 320° − 180°, or 140°.

From the preceding explanation, it is seen that the back azimuth of a line can be found from its forward azimuth as follows: If the azimuth of the line is less than 180°, add 180° to find the back azimuth. When the azimuth of the line is greater than 180°, subtract 180° to obtain the back azimuth.

7-8 Bearing of a Line

The bearing of a line also gives the direction of the line with respect to the reference meridian. Unlike an azimuth, which is always an angle measured in a definite direction from a definite half of the meridian, a bearing angle is never greater than 90°. The bearing states whether the angle is measured from the north or the south and also whether the angle is measured toward the east or toward the west. For

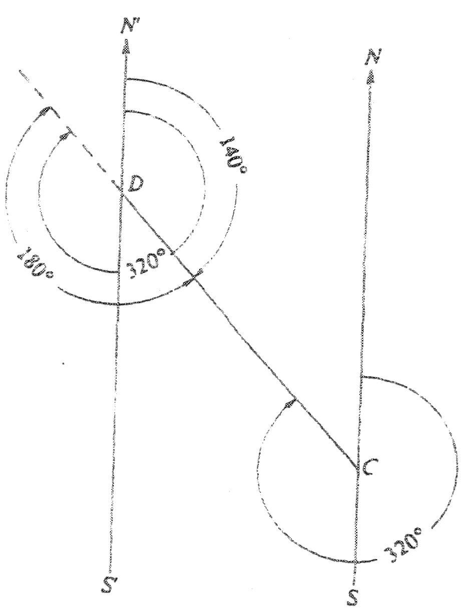


FIGURE 7-3 Azimuth and back azimuth.

example, if a line has a bearing of S 35° E (called south 35° east), the bearing angle 35° is measured from the south meridian eastward. Figure 7-4 shows four lines, the origin in each case being point *O*. Each line lies in a different quadrant. The bearing of *OA* is N 70° E; that of *OB* is S 35° E; that of *OC* is S 55° W; and that of *OD* is N 30° W.

It is apparent from Fig. 7-4 that the bearing angle of a line must be between 0° and 90°. A stated bearing is a *true* bearing, a *magnetic* bearing, an *assumed* bearing, or a *grid* bearing, according to whether the reference meridian is true, magnetic, assumed, or grid. In land surveying and in the conveyance of title to property, references are made to maps, notes, and plats of previous surveys that are recorded with the various counties throughout the United States. When reference is made to a bearing in such a previous survey, the term *record bearing* is used. If reference is made to a property deed, the term *deed bearing* is used. The terms *deed bearing* and *record bearing* are commonly used interchangeably.

7-9 Back Bearing

The bearing of a line in the direction in which a survey containing several lines is progressing is called the *forward* bearing, whereas the bearing of the line in the direction opposite to that of progress is the *back* bearing. The back bearing can be obtained from the forward bearing by simply changing the letter N to S or S to N and also changing E to W or W to E. In Fig. 7-5 the bearing of the line *AB* is N 68° E, and the bearing of *BA*, which is in the reverse direction is S 68° W.

7-10 Relation Between Azimuths and Bearings

To simplify computations based on survey data, bearings may be converted to azimuths and azimuths may be converted to bearings. The instances when such conversion is convenient will become apparent in later chapters. The conversion itself is quite simple.

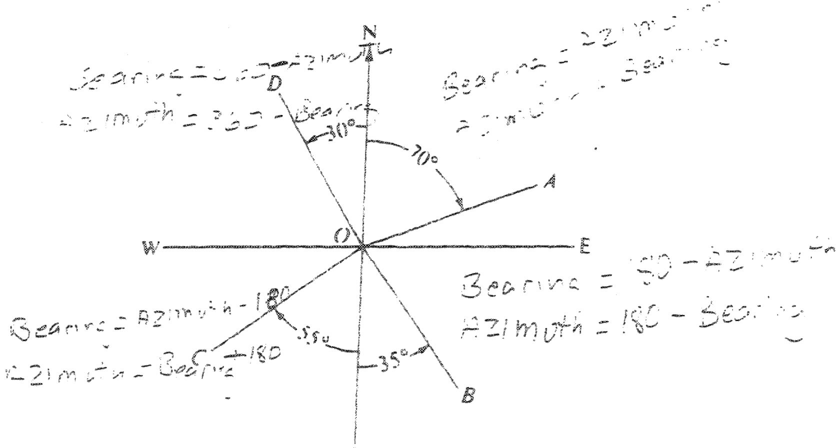


FIGURE 7-4 Bearings.

An inspection of Fig. 7-1 will show that the line *OA*, whose azimuth from north is 70°, lies in the northeast quadrant since the angle eastward from the meridian is less than 90°. Furthermore it is apparent that the bearing angle and the azimuth are identical. Therefore the bearing of *OA* is N 70° E. The line *OB* is 145° from the north meridian. It lies south of a due-east line and is therefore in the southeast quadrant. The problem in this case is to determine the angle from the south meridian. Since the north meridian and the south meridian are 180° apart, the problem is solved by subtracting the azimuth, or 145°, from 180° to arrive at the bearing angle, which is 35°. Therefore the bearing of the line *OB* is S 35° E. The line *OC* is 235° from the north meridian in a clockwise direction and is beyond the south meridian in a westerly direction by an angle of 235° − 180°, or 55°. Therefore if the azimuth is 235°, the bearing is S 55° W. For the line *OD* the angle from the north meridian is 330° in a clockwise direction. The angle from the north meridian in a counterclockwise or westerly direction is 360° − 330°, or 30°. The line *OD* lies in the northwest quadrant, and its bearing is N 30° W.

The rules to observe in converting from azimuths to bearings are very quickly established in a person's mind after the rules have been put to practice a few times. They are as follows: (1) If an azimuth from north is between 0° and 90°, the line is in the northeast quadrant, and the bearing angle is equal to the azimuth; (2) if an azimuth from north is between 90° and 180°, the line is in the southeast quadrant, and the bearing angle is 180° minus the azimuth; (3) if the azimuth from north is between 180° and 270°, the line is in the southwest quadrant, and the bearing angle is the azimuth minus 180°; (4) if the azimuth from north is between 270° and 360°, the line is in the northwest quadrant, and the bearing angle is 360° minus the azimuth.

To convert from bearings to azimuths, it is only necessary to reverse the foregoing rules, and the computations should be made with the same mental ease. The azimuth of a line in the northeast quadrant is equal to the bearing angle; that of a line in the southeast quadrant is 180° minus the bearing angle; that of a line in the southwest quadrant is 180° plus the bearing angle; that of a line in the northwest quadrant is 360° minus the bearing angle. As an example, the azimuth of a line whose bearing is N 77° 43′ 16″ W is 282° 16′ 44″.

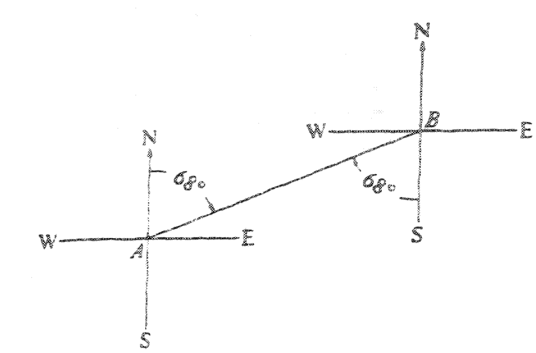


FIGURE 7-5 *Bearing and back bearing*

7-11 Magnetic Compass

Since a freely suspended magnetized needle will lie in the magnetic meridian, the direction of a line can be determined with respect to the needle, and thus to the magnetic meridian, by measuring the angle between the line and the needle. The magnetic compass is constructed so as to allow a needle to swing freely on a pivot when in use, and to allow a line of sight to be directed from the occupied point to a terminal point. As shown in Fig. 7-6, a graduated circle is rotated as the line of sight is rotated. The north-seeking end of the compass needle is read against the circle to obtain the angle between the magnetic meridian and the line of sight.

The circle and the needle are encased in a metal compass box and are covered with a glass plate. The line of sight normally is fixed in line with the zero mark or the north graduation on the circle. Thus, if a line of sight is directed along the north

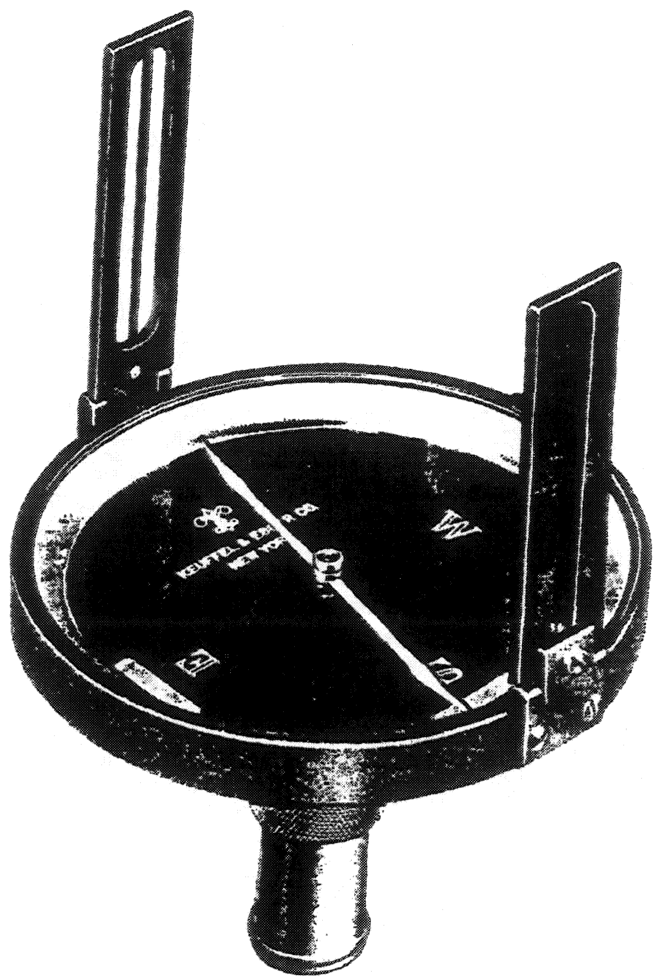


FIGURE 7-6 Surveyor's compass.

magnetic meridian, the needle will point to the zero mark or to the north graduation. As the line of sight is turned clockwise from magnetic north, the needle remains in the magnetic meridian, but the graduated circle is turned clockwise through the corresponding angle. When the line of sight is turned exactly 90° east of north, then the letter E is brought opposite the north end of the compass needle. The circle thus indicates that the magnetic bearing is due east, as it should. If the line of sight is turned exactly 180° from north, the letter S is brought opposite the north end of the needle, and the magnetic bearing of the line of sight is shown to be due south.

The surveyor's compass (Fig. 7-6) is the instrument that was used in the past to run surveys of reasonable accuracy. Its main part is the compass box containing the graduated circle between 4 and 6 in. in diameter, the magnetic needle, one or two level bubbles to allow the circle to be brought horizontal, long sight vanes to allow fairly accurate pointings along rather steeply inclined lines, and a screw for lifting the needle off the pivot and against the glass cover plate. The compass box is fitted with a spindle that rotates in a socket. The socket fits in a leveling head consisting of a ball-and-socket joint, and the whole compass assembly is leveled about the ball-and-socket against friction applied by a clamping nut. The leveling head is fastened to a tripod or to a staff and is used in the field in this fashion. The circle is usually graduated in half degrees. The sight vanes are aligned so that the line of sight passes through the north graduation on the circle.

The transit compass is similar to the surveyor's compass in all respects save for the method of leveling and for the line of sight. The compass box sets in the center of the horizontal limb of the transit, astride of which are the standards that hold the telescope (see Fig. 4-8). The method of leveling the transit, and thus the compass box, was described in Section 4-14. The line of sight for the transit compass is the collimation line of the telescope defined by the center of the objective lens and the intersection of the cross hairs. The telescope of the transit is capable of being raised or depressed for sighting along inclined lines. The letter N on the compass circle is normally under the objective end of the telescope when the telescope is in the direct position (see Section 4-17). The letter S is under the eyepiece. Therefore this arrangement keeps the north graduation directed along the line of sight just as in the case of the surveyor's compass.

7-12 Determining Directions with a Magnetic Compass

When using a surveyor's compass or a transit, the observer occupies one end of the line, centers the instrument over the point, and levels the instrument. He releases the clamp holding the needle, allowing it to rest on the pivot. He then directs the line of sight toward a point at the other end of the line and brings it on that point. When the needle comes to rest, he reads the circle at the north end of the needle to obtain the bearing of the line from the occupied point to the point sighted on. He then clamps the needle before disturbing the instrument.

To obtain the back magnetic bearing as a check, he occupies the second point and sights back to the first point, reading the north end of the needle as before. The bearing angles should show reasonable agreement, and the letters N and S as well as E and W should be reversed on the back bearing. This check should be made before leaving the point.

7-13 Magnetic Declination

The magnetic poles do not coincide with the poles defined by the earth's rotational axis, and certain irregularities in the earth's magnetic field cause local and regional variations in the direction of the needle. Therefore, except in a very narrow band around the earth, the magnetic needle does not point in the direction of true north. It some areas the needle points east of true north; in other areas the needle points west of true north. The amount and direction by which the magnetic needle is off the true meridian is called the *magnetic declination* (formerly called *variation*). Declination is positive or plus when the needle points east of true north, and it is negative or minus when the needle points west of true north. The declination varies from +23° in the state of Washington to –22° in the state of Maine; the total range over the United States is 45°.

The declination in a given vicinity can be obtained by observing the magnetic bearing of a line of known true azimuth or bearing.

EXAMPLE 7-1 The magnetic bearing of a line of known true azimuth is observed as S55° 40′E. The true azimuth of this line is 109° 05′. What is the magnetic declination at the point?

Solution: The magnetic azimuth according to Section 7-10 is 124° 20′. As shown in Fig. 7-7, the difference is 15° 15′, and the declination is 15° 15′W.

7-14 Horizontal Angles from Compass Bearings

An angle at a point can be obtained, within the limits of accuracy of compass readings, by observing the bearings to the backsight station and the foresight station. The resulting angle is free from the effects of the declination and of any local attraction affecting the compass needle since both sights are referred to the compass needle.

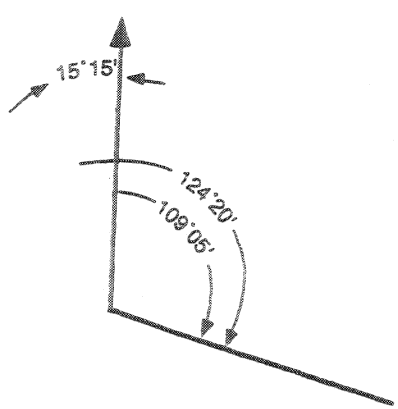


FIGURE 7-7 Magnetic declination.

EXAMPLE 7-2 The following bearings were observed at the points in the four-sided traverse shown in Fig. 7-8. Compute and adjust the angles.

Line	Observed Mag Bearing	Unadjusted Angles	Adjusted Angles
AD	S 77° 00′ E		
		(180° − 77° 00′) + 12° = 115° 00′	114° 45′
AB	N 12° 00′ W		
BA	S 12° 00′ E		
		82° 30′ − 12° 00′ = 70° 30′	70° 15′
BC	S 82° 30′ E		
CB	N 82° 00′ W		
		180° − (82° 00′ + 20° 00′) = 78° 00′	77° 45′
CD	S 20° 00′ W		
DC	N 20° 00′ E		
		20° + 77° 30′ = 97° 30′	97° 15′
DA	N 77° 30′ W		
		361° 00′	360° 00′

Solution: The calculations are shown in the accompanying table. The student should refer to Fig. 7-8 to follow these calculations. Since the angles do not total $(n-2)180°$, they are adjusted equally by subtracting 15′ from each angle.

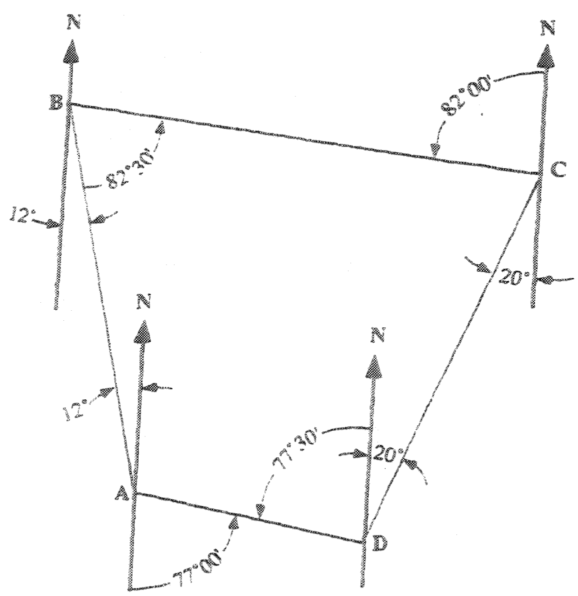


FIGURE 7-8 Horizontal angles from compass bearings.

PROBLEMS

7-1. Convert the following bearings to azimuths: **(a)** S 15° 25′ 20″ E, **(b)** N 18° 16′ 30″ E, **(c)** N 18° 16′ 30″ W, **(d)** S 27° 20′ 15″ W, **(e)** S 88° 16′ 40″ E.

7-2. Convert the following bearings to azimuths: **(a)** N 45° 27′ 45″ W, **(b)** N 27° 20′ 35″ E, **(c)** S 27° 20′ 35″ E, **(d)** S 77° 20′ 15″ W, **(e)** N 89° 10′ 15″ W.

7-3. Convert the following azimuths to bearings: **(a)** 106° 22′ 10″, **(b)** 57° 18′ 50″, **(c)** 327° 14′ 20″, **(d)** 267° 53′ 15″, **(e)** 178° 14′ 20″.

7-4. Convert the following azimuths to bearings: **(a)** 20° 12′ 30″, **(b)** 90° 04′ 50″, **(c)** 166° 14′ 40″, **(d)** 270° 14′ 15″, **(e)** 354° 26′ 15″.

7-5. The true azimuth of a line is 187° 10′. The observed magnetic bearing of the line is S 2° 15′ E. What is the magnetic declination of the point of observation?

7-6. In a five-sided traverse, ABCDE, the following magnetic bearings were observed:

Line	Observed Magnetic Bearing
AE	S 7° 00′ W
AB	N 75° 30′ E
BA	S 75° 30′ W
BC	S 45° 30′ W
CB	N 46° 00′ E
CD	S 15° 00′ E
DC	N 14° 30′ W
DE	N 82° 30′ W
ED	S 82° 30′ E
EA	N 6° 30′ E

Compute the interior angles in the traverse and adjust them equally.

8 Traverse Surveys and Computations

8-1 Traverse

A traverse is a series of connected lines of known length related to one another by known angles. The lengths of the lines are determined by direct measurement of horizontal distances, by slope measurement, or by indirect measurement based on the methods of tacheometry. These methods are discussed in Chapters 2 and 14. The angles at the traverse stations between the lines of the traverse are measured with the instruments discussed in Chapter 4. They can be interior angles, deflection angles, or angles to the right, which are discussed in Chapter 6.

The results of field measurements related to a traverse will be a series of connected lines whose lengths and azimuths, or whose lengths and bearings, are known. The lengths are horizontal distances; the azimuths or bearings are either true, magnetic, assumed, or grid.

A completely different method of traversing can be performed using an inertial system. This system incorporates a gyro-stabilized platform containing three accelerometers oriented at right angles to one another, a precise clock, and an onboard computer. The accelerometers sense accelerations in the north–south, east–west, and vertical directions from which displacements in these three directions are computed.

The satellite constellation of the Global Positioning System (GPS) is used in traversing by the method referred to as *kinematic GPS surveying* and is described in detail in Chapter 10.

In general, traverses are of two classes. One of the first class is an open traverse. It originates either at a point of known horizontal position with respect to a horizontal datum or at an assumed horizontal position, and terminates at an unknown horizontal position. A traverse of the second class is a closed traverse, which can be described in any one of the following three ways: (1) It originates at an assumed horizontal position and terminates at that same point; (2) it originates at a known horizontal position with respect to a horizontal datum and terminates at that same point; (3) it originates at a known horizontal position and terminates at another known horizontal position. A known horizontal position is defined by its geographic latitude and longitude. by its Y and X coordinates on a grid system, or by its location on or in relation to a fixed boundary.

Traverse surveys are made for many purposes and types of projects, some of which follow:

- To determine the positions of existing boundary markers
- To establish the positions of boundary lines
- To determine the area encompassed within the confines of a boundary
- To determine the positions of arbitrary points from which data may be obtained for preparing various types of maps, that is, to establish *control* for mapping
- To establish ground control for photogrammetric mapping
- To establish control for gathering data regarding earthwork quantities in railroad, highway, utility, and other construction work
- To establish control for locating railroads, highways, and other construction work

8-2 Open Traverse

An open traverse is usually run for exploratory purposes. There are no arithmetical checks on the field measurements. Since the figure formed by the surveyed lines does not close, the angles cannot be summed to a known quantity, for example, the angles in a plane triangle sum to 180°. None of the positions of the traverse stations can be verified, since no known or assumed position is included except that of the starting station. To strengthen an open traverse, that is, to render it more reliable, several techniques may be employed. Each distance can be measured in both directions and can be roughly checked by using the stadia hairs of the theodolite (see Chapter 14). The measurements of the angles at the stations can be repeated by using the methods of Chapters 4 and 6 and checked approximately by observing magnetic bearings. The directions of the lines can be checked by observing the sun or the stars to determine true azimuths or bearings of selected lines in the traverse. An open traverse should not be run for any permanent project or for any of the projects indicated in Section 8-1 because it does not reveal mistakes or errors and the results are always open to doubt.

8-3 Closed Traverse

A traverse that closes on itself immediately affords a check on the internal accuracy of the measured angles, provided that the angle at each station has been measured. As will be discussed later, a traverse that closes on itself gives an indication of the consistency of measuring distances as well as angles by affording a check on the position closure of the traverse. Unless astronomical observations for azimuths (see Chapter 12) have been made at selected stations of a traverse that closes on itself, the only provision for verifying the directions of the lines is that afforded by the angular closure. In this type of traverse, there is no check on the systematic errors introduced into the measurement of lengths. Therefore, when this type of traverse is executed for a major project, the measuring apparatus must be carefully calibrated to determine the systematic errors and to eliminate them.

A traverse that originates at a known position and closes on another known position is by far the most reliable, because a check on the position of the final point checks both the linear and angular measurements of the traverse. When a point of known position is referred to, it is understood that such a point has been located by procedures as precise as, or more precise than, those used in the traverse being executed. These procedures are the methods of either traversing, triangulation, trilateration, or GPS surveying. Triangulation and trilateration are the subjects of Chapter 9.

8-4 Interior-Angle Traverse

An interior-angle traverse is shown in Fig. 8-1. The azimuth or bearing of the line *AP* is known. The lengths of the traverse lines are measured to determine the horizontal distances. With the transit or theodolite at *A*, the angle at *A* from *P* to *B* is measured to determine the azimuth of line *AB*. The angle at *A* from *E* to *B* is also measured, as this is one of the interior angles in the figure. The instrument is then set up at *B*, *C*, *D*, and *E* in succession, and the indicated angles are measured. The notes for the angles are recorded as indicated in Chapter 6. Had the azimuth of one of the traverse sides been known, there would have been no necessity for measuring the angle at *A* from *P* to *B*.

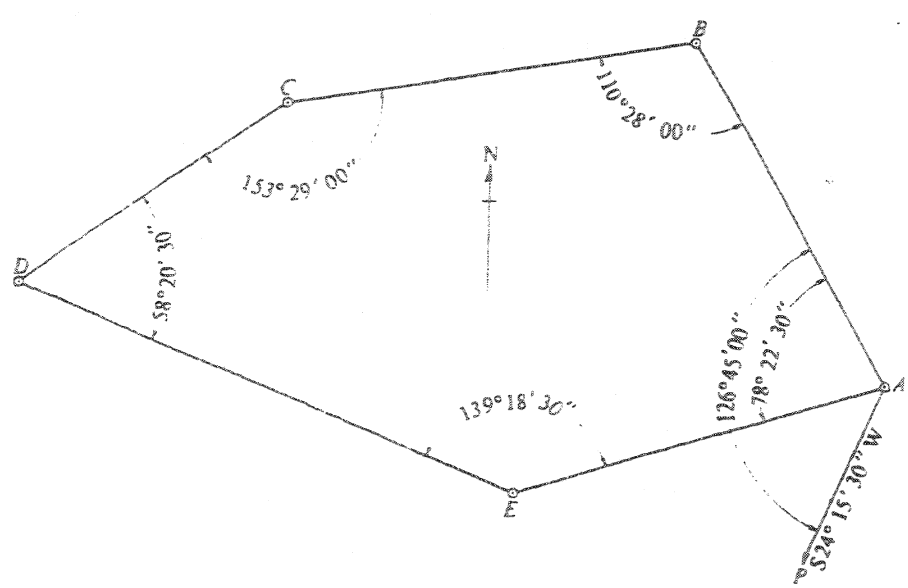


FIGURE 8-1 Interior-angle traverse.

TABLE 8-1 Adjustment of Angles in an Interior-Angle Traverse

Station	Measured Angle	Correction	Adjusted Angle
A	78° 22′ 30″	+18″	78° 22′ 48″
B	110° 28′ 00″	+18″	110° 28′ 18″
C	153° 29′ 00″	+18″	153° 29′ 18″
D	58° 20′ 30″	+18″	58° 20′ 48″
E	139° 18′ 30″	+18″	139° 18′ 48″
	539° 58′ 30″		540° 00′ 00″
	−540° 00′ 00″		
	closure = −1′30″		

To test the internal angular closure, the interior angles are added and their sum compared to $(n-2)180°$, which in this example is $(5-2)180° = 540°$. The total angular error, or the closure, is 1′ 30″. In the tabulation in Table 8-1 the measured angles are adjusted by assuming that the angular error is of the same amount at each station. This assumption may not be valid, because an error in angular measurement, all other things being equal, will increase as the lengths of the adjacent sides decrease. In the traverse of Fig. 8-1 the interior angles are presumed to have been measured to the nearest 30″. The manner of adjusting the 1′ 30″ closure can be somewhat arbitrary in this instance. For example, instead of applying an 18″ correction to each angle, the discrepancy could be distributed by correcting the angles at *A*, *B*, and *C* by 30″ each, or by correcting the angles at *A*, *C*, and *D* by 30″ each.

After the adjusted angles are computed, they should always be added to see whether their sum is, in fact, the proper amount. A mistake in arithmetic either in adding the measured angles or in applying the corrections will become apparent.

The azimuth of the line *AP* in Fig. 8-1 is known, and the azimuths of all the traverse sides can be determined by using the measured angle at *A* from *P* to *B* and the adjusted interior angles in the closed figure. Since the bearing of *AP* is S 24° 15′ 30″ W, the azimuth of *AP* reckoned from north is 204° 15′ 30″. As indicated in Fig. 8-2, the azimuth of *AB* is equal to the azimuth of *AP* plus the angle at *A* from *P* to *B*, or 204° 15′ 30″ + 126° 45′ 00″ = 331° 00′ 30″. The azimuths of the other lines are computed systematically by applying the adjusted angle at each station to the azimuth of each backsight line in turn. To determine the azimuth of *BC*, compute the azimuth of *BA* by subtracting 180° from the azimuth of *AB*, and then add the adjusted angle at *B* from *A* to *C*. A similar procedure is adopted at each station (see Table 8-2).

The azimuths of the lines can also be computed by applying the angles at *A*, *E*, *D*, *C*, and *B* in that order. Then each interior angle would be ***subtracted*** from the azimuth of the proper back line. This is apparent from a study of Fig. 8-1.

The bearing of each line, if desired, is determined from its azimuth. Since the conversion from azimuths to bearings is easily made, it is unwise to attempt to compute the bearing of each line directly by applying the proper angle in the traverse to the bearing of the adjacent line.

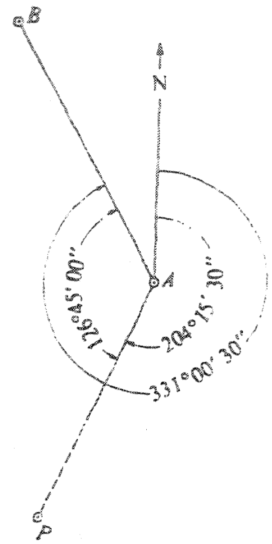


FIGURE 8-2 Determining azimuth of line *AB*.

TABLE 8-2 Computation of Azimuths and Bearings Using Adjusted Interior Angles

Line	Azimuth	Bearing
AP	204° 15′ 30″	S 24° 15′ 30″ W
	(+ ∠*A*) + 126° 45′ 00″	
AB	331° 00′ 30″	N 28° 59′ 30″ W
BA	151° 00′ 30″	
	(+ ∠*B*) + 110° 28′ 18″	
BC	261° 28′ 48″	S 81° 28′ 48″ W
CB	81° 28′ 48″	
	(+ ∠*C*) + 153° 29′ 18″	
CD	234° 58′ 06″	S 54° 58′ 06″ W
DC	54° 58′ 06″	
	(+ ∠*D*) + 58° 20′ 48″	
DE	113° 18′ 54″	S 66° 41′ 06″ E
ED	293° 18′ 54″	
	(+ ∠*E*) + 139° 18′ 48″	
EA	432° 37′ 42″	
EA	72° 37′ 42″	N 72° 37′ 42″ E
AE	252° 37′ 42″	
	(+ ∠*A*) + 78° 22′ 48″	
AB	331° 00′ 30″ check	N 28° 59′ 30″ W

In a traverse of n sides or stations, the closure of the measured angles should not exceed the least count of the vernier or scale of the instrument times the square root of n. With care in sighting, in centering the instrument, and in reading the verniers or scales, the angular closure can be expected to be half this allowable amount. Of course, with few traverse stations, this precision may not be obtainable since the opportunities for random errors to compensate are few.

One very important point to observe in the preceding example of an interior-angle traverse is that there is no check on the angle at A from P to B. If this angle is in error, then the error affects the azimuth of each line in the traverse. To avoid the possibility of making a mistake in measuring this angle, the clockwise angle at A from B to P should also be measured. This measurement immediately affords a check at station A since the angle at A from P to B and the angle at A from B to P should total 360°. If the azimuth of one of the sides of the traverse were known, then this uncertainty would have been avoided, because the test of the accuracy in this case is simply that the sum of the interior angles should equal $(n-2)180°$.

8-5 Deflection-Angle Traverse

A deflection-angle traverse that originates at station 0 + 00 and closes on station 22 + 20 is shown in Fig. 8-3. The azimuth of the line FG is fixed as 196° 35′ and the azimuth of EM is fixed as 306° 43′, these directions having been established by previous surveys. The traverse is made to fit between these two fixed azimuths. After the deflection angles and the lengths of the several courses have been measured, the check on angular closure is made by carrying azimuths through the traverse by applying the deflection angles. In Fig. 8-4 the azimuth of GF is 16° 35′, obtained by

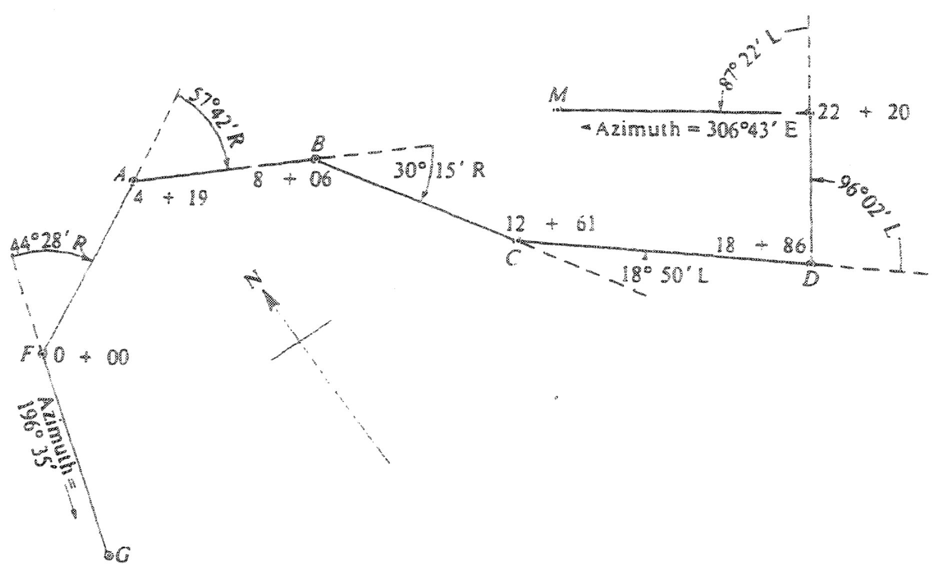


FIGURE 8-3 *Deflection-angle traverse between fixed azimuths.*

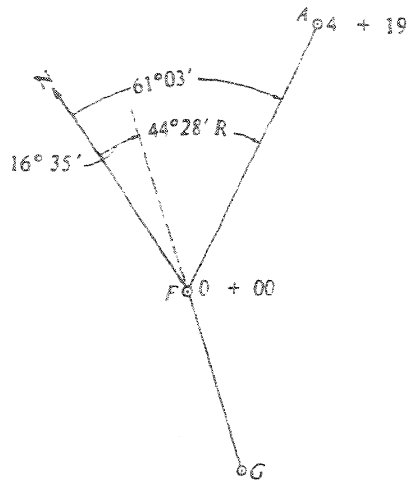


FIGURE 8-4 Determining azimuth of line *FA*.

subtracting 180° from the azimuth of *FG*. So the azimuth of the line *FA* is 16° 35′ plus 44° 28′, which is the right deflection angle at *F* from *G* to *A*. The addition gives 61° 03′ as the azimuth of *FA*. Now to determine the azimuth of the next line *AB*, the right deflection angle at *A* from *F* to *B* is added to the azimuth of *FA* prolonged. This sum is 61° 03′ + 57° 42′ = 118° 45′. Observe that no back azimuths need be computed as was the case for the interior-angle traverse. To compute an azimuth by using a deflection angle, simply add a right deflection angle to the *forward* azimuth of the backsight to obtain the *forward* azimuth of the foresight; or subtract a left deflection angle from the *forward* azimuth of the backsight to obtain the *forward* azimuth of the foresight. A computation of azimuths in a deflection-angle traverse is given in Table 8-3.

In the example in Fig. 8-3, the computed azimuth of *EM* failed to check by 3′. Since six deflection angles were measured, the correction to each angle is 30″. Instead of applying this correction to each angle and recomputing the azimuths, the azimuths themselves are adjusted. The azimuth of *FA* receives a 30″ correction, since this azimuth was obtained by considering only one measured angle; the azimuth of *AB* receives a 1′ 00″ correction, since this azimuth was obtained by using two angles, and so on. The correction to the last azimuth is 6 × 30″ = 3′ 00″, since this azimuth was obtained by using all six deflection angles.

Figure 8-5 illustrates deflection-angle traverse that closes on the point of origin at *L*. The bearing of the line *OP* is known to be N 66° 02′ W. Before the bearings of the remaining sides are computed, the angles must be adjusted so that the difference between the sum of the right deflection angles and the sum of the left deflection angles is 360°. It is found that the sum of the right deflection angles must be reduced and the sum of the left deflection angles must be increased. The correction in this case is distributed equally among all of the angles. Had the angular closure been, for example, only 01′, then 30″ could have logically been added to the angle at *N* and subtracted from the angle at *Q*, since these two angles are

TABLE 8-3 Computation of Azimuths and Bearings Using Deflection Angles

Line	Azimuth	Correction	Adjusted Azimuth	Adjusted Bearing
CF	16° 35′	fixed		N 16° 35′ 00″ E
	(+ ∠*F*) + 44° 28′			
FA	61° 03′	−0′ 30″	61° 02′ 30″	N 61° 02′ 30″ E
	(+ ∠*A*) + 57° 42′			
AB	118° 45′	−1′ 00″	118° 44′ 00″	S 61° 16′ 00″ E
	(+ ∠*B*) + 30° 15′			
BC	149° 00′	−1′ 30″	148° 58′ 30″	S 31° 01′ 30″ E
	(− ∠*C*) − 18° 50′			
CD	130° 10′	−2′ 00″	130° 08′ 00″	S 49° 52′ 00″ E
	(− ∠*D*) − 96° 02′			
DE	34° 08′	−2′ 30″	34° 05′ 30″	N 34° 05′ 30″ E
DE	394° 08′			
	(− ∠*E*) − 87° 22′			
EM	306° 46′	−3′ 00″	306° 43′ 00″	N 53° 17′ 00″ W
	306° 43′	fixed		
	closure = + 03′			

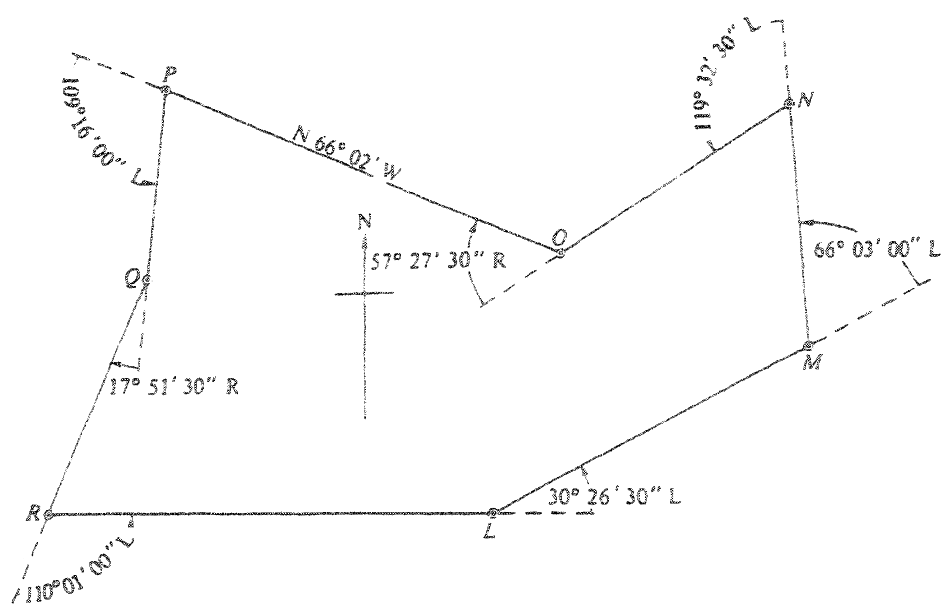


FIGURE 8-5 Deflection-angle traverse that closes on point of origin (adjusted angles are shown).

TABLE 8-4 Adjustment of Deflection Angles

Station		Deflection Angle	Correction		Adjusted Deflection Angle
L		30° 26′ 00″ L	+30″		30° 26′ 30″ L
M		66° 02′ 30″ L	+30″		66° 03′ 00″ L
N		119° 32′ 00″ L	+30″		119° 32′ 30″ L
O		57° 28′ 00″ R	−30″		57° 27′ 30″ R
P		109° 15′ 30″ L	+30″		109° 16′ 00″ L
Q		17° 52′ 00″ R	−30″		17° 51′ 30″ R
R		110° 00′ 30″ L	+30″		110° 01′ 00″ L
	Σ right	75° 20′ 00″		Σ right	75° 19′ 00″
	Σ left	435° 16′ 30″		Σ left	435° 19′ 00″
	difference	359° 56′ 30″			360° 00′ 00″ check
	closure	3′ 30″			

formed by the shortest lines of sight. The tabulation of Table 8-4 shows this adjustment of deflection angles.

Bearings of the sides of the traverse are computed by first converting the bearing of *OP* to an azimuth, then applying the adjusted deflection angles to obtain azimuths, and finally converting the azimuths to bearings as shown in Table 8-5.

The angular closure of a deflection-angle traverse should be no more than the least count of the vernier of the instrument times the square root of the number of angles in the traverse. A deflection angle should never be measured without double

TABLE 8-5 Computation of Azimuths and Bearings Using Adjusted Deflection Angles

Line	Azimuth	Bearing
OP	293° 58′ 00″	N 66° 02′ 00″ W
	(− ∠*P*) − 109° 16′ 00″	
PQ	184° 42′ 00″	S 4° 42′ 00″ W
	(+ ∠*Q*) + 17° 51′ 30″	
QR	202° 33′ 30″	S 22° 33′ 30″ W
	(− ∠*R*) − 110° 01′ 00″	
RL	92° 32′ 30″	S 87° 27′ 30″ E
	(− ∠*L*) − 30° 26′ 30″	
LM	62° 06′ 00″	N 62° 06′ 00″ E
LM	422° 06′ 00″	
	(− ∠*M*) − 66° 03′ 00″	
MN	356° 03′ 00″	N 3° 57′ 00″ W
	(− ∠*N*) − 119° 32′ 30″	
NO	236° 30′ 30″	S 56° 30′ 30″ W
	(+ ∠*O*) + 57° 27′ 30″	
OP	293° 58′ 00″ check	N 66° 02′ 00″ W

centering the instrument, because the error caused by the line of sight not being normal to the horizontal axis of the instrument may be too large to be tolerated.

8-6 Angle-to-the-Right Traverse

Either an open traverse or a closed traverse can be executed by measuring angles to the right. The method of measuring the angles is described in Section 6-3. The method of computing azimuths from a given fixed azimuth is similar to that employed in an interior-angle traverse. A forward azimuth is always obtained by *adding* the angle to the right to the azimuth of the backsight. Angles to the right are always employed when a traverse is executed with a direction instrument.

Figure 8-6 is an illustration of an angle-to-the-right traverse executed by use of a direction instrument. Station *P* is occupied, a backsight is taken on station *T*, and the circle is read. A foresight is taken on station *Q*, and the circle is read. This procedure is repeated at each station along the traverse, each backsight and foresight being observed with the telescope both direct and reversed. Notes for a 10″ direction instrument are given in Table 8-6.

Assume that the azimuth of *PT* is known to be 88° 20′ 10″. The azimuth of the line *PQ* is the azimuth of *PT* plus the angle to the right at *P*. The angle to the right at *P* from *T* to *Q* can be obtained by subtracting the mean circle reading to *T* from the mean circle reading to *Q*. This angle, as computed from notes, is 92° 16′ 00″ − 51° 54′ 50″ = 40° 21′ 10″. Then the azimuth of *PQ* is 88° 20′ 10″ + 40° 21′ 10″ = 128° 41′ 20″. This computation can be simplified if the mean circle reading to *Q* is added to the azimuth of *PT*, and then the mean circle reading to *T* is subtracted from the sum. In this manner the circle reading for the foresight is always added and the circle reading for the backsight is always subtracted.

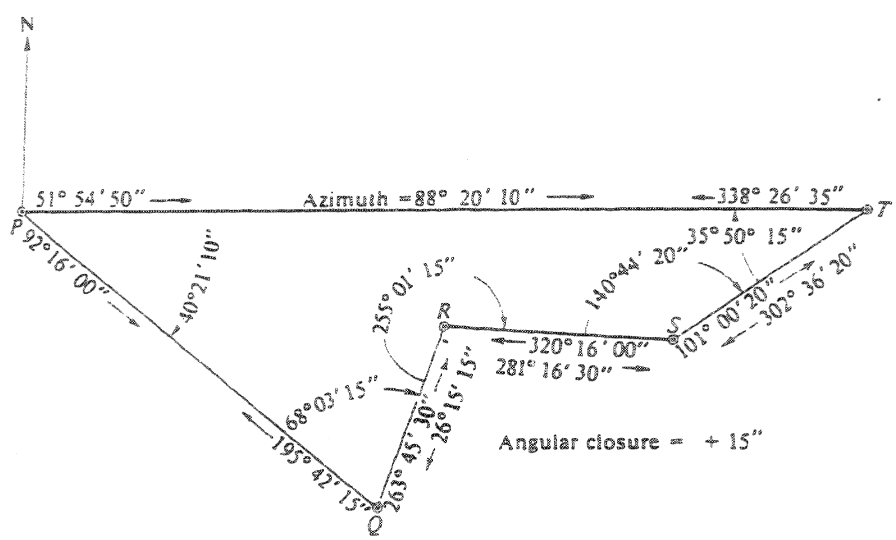


FIGURE 8-6 Angles-to-the-right from direction instrument readings.

TABLE 8-6 Notes for Measuring Angles to the Right When Using a Direction Instrument

Station		D or R	Reading	Mean
At *P*	*T*	D	51° 54′ 50″	51° 54′ 50″
		R	231° 54′ 50″	
	Q	D	92° 15′ 50″	92° 16′ 00″
		R	272° 16′ 10″	
At *Q*	*P*	D	195° 42′ 10″	195° 42′ 15″
		R	15° 42′ 20″	
	R	D	263° 45′ 20″	263° 45′ 30″
		R	83° 45′ 40″	
At *R*	*Q*	D	26° 15′ 10″	26° 15′ 15″
		R	206° 15′ 20″	
	S	D	281° 16′ 30″	281° 16′ 30″
		R	101° 16′ 30″	
At *S*	*R*	D	320° 16′ 00″	320° 16′ 00″
		R	140° 16′ 00″	
	T	D	101° 00′ 10″	101° 00′ 20″
		R	281° 00′ 30″	
At *T*	*S*	D	302° 36′ 10″	302° 36′ 20″
		R	122° 36′ 30″	
	P	D	338° 26′ 30″	338° 26′ 35″
		R	158° 26′ 40″	

The computation is shown in Table 8-7 as it would be made without the aid of a calculating machine designed for adding and subtracting angles on the sexagesimal system. If such a calculating machine is used, only the forward azimuth of each line of the traverse need be recorded on the computation sheet. The resulting azimuths are then adjusted to the fixed azimuth, and bearings may be obtained from the adjusted azimuths.

8-7 Traverse-by-Azimuth Method

In running a traverse for the purpose of establishing a lower order of control for mapping and for locating the positions of ground objects with respect to such a supplementary traverse, the transit or theodolite can be handled so that the clockwise circle reading at all times indicates the azimuth of the line of sight. This procedure eliminates the need for computing azimuths from interior angles, deflection angles, or angles to the right.

Figure 8-7 shows a portion of a supplementary traverse in which the line *DE* has an azimuth of 96° 22′. This may be a true, magnetic, assumed, or grid azimuth. The instrument is set up at station *E*, and the clockwise circle is set to read the azimuth of the line *ED*, which is 276° 22′. A lower motion is used to take a backsight along the line *ED*. The theodolite and the horizontal circle are now oriented. In other words, when any sight is taken by an upper motion, the clockwise circle reading will always indicate the azimuth of the line of sight. If a pointing is made

TABLE 8-7 Computations of Azimuths Using Angle-to-the Right Notes

Line	Azimuth	Correction	Adjusted Azimuth
PT	88° 20′ 10″ fixed		
	+ 92° 16′ 00″		
	180° 36′ 10″		
	− 51° 54′ 50″		
PQ	128° 41′ 20″	−3″	128° 41′ 17″
QP	308° 41′ 20″		
	+ 263° 45′ 30″		
	572° 26′ 50″		
	− 195° 42′ 15″		
QR	376° 44′ 35″		
QR	16° 44′ 35″	−6″	16° 44′ 29″
RQ	196° 44′ 35″		
	+ 281° 16′ 30″		
	478° 01′ 05″		
	− 26° 15′ 15″		
RS	451° 45′ 50″		
RS	91° 45′ 50″	−9″	91° 45′ 41″
SR	271° 45′ 50″		
	+ 101° 00′ 20″		
	372° 46′ 10″		
	− 320° 16′ 00″		
ST	52° 30′ 10″	−12″	52° 29′ 58″
TS	232° 30′ 10″		
	+ 338° 26′ 35″		
	570° 56′ 45″		
	− 302° 36′ 20″		
TP	268° 20′ 25″	−15″	268° 20′ 10″
PT	88° 20′ 25″ fixed		
	88° 20′ 10″		
	closure + 15″		

on station *F* and the clockwise circle reads 68° 02′, this is the azimuth of the line *EF* and is recorded as such.

When the transit is set up at *F*, the circle is oriented by setting the azimuth of *FE*, which is 248° 02′, on the clockwise circle and backsighting along the line *FE* by a lower motion. A sight taken on point *P* by an upper motion will give the azimuth of the line *FP* directly on the clockwise circle. The readings to *P*, *Q*, and *G* are shown to be, respectively, 57° 25′, 192° 52′, and 113° 10′.

The advantages of executing a traverse by observing azimuths are that the transit or theodolite is allowed to do the work of adding and subtracting angles, and that the field notes are easily reduced to map form by having all lines related to the same meridian directly in the notes.

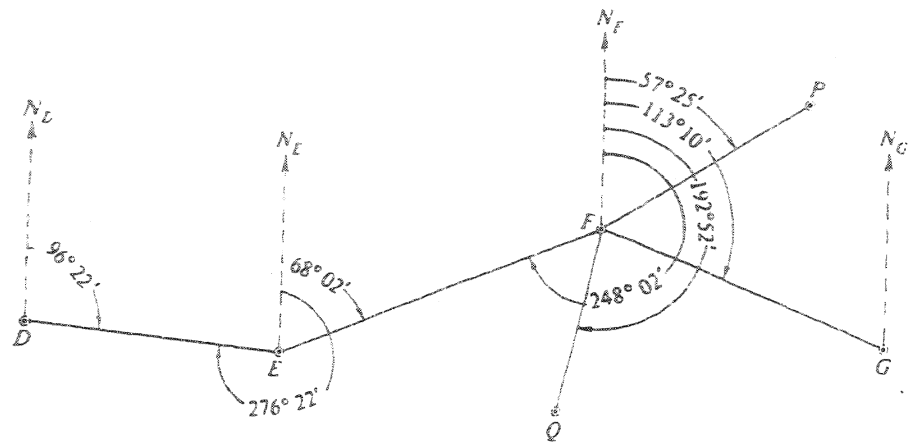


FIGURE 8-7 Traverse by azimuth method.

The disadvantage of such a procedure is in not realizing the benefit of double centering, which eliminates instrumental errors and which makes mistakes in reading the circle quite obvious. The purpose of the traverse, however, is usually such that small errors are of little consequence (see Section 14-8).

If a traverse is to be run on magnetic azimuths and the directions of the lines are to be consistent with one another, the initial setup determines the specific magnetic meridian to which all other lines are referred. To orient on the magnetic meridian, set the clockwise circle to read zero, unclamp the compass needle, loosen the lower clamp, and rotate the transit until the compass needle points to the north point of the compass circle. Tighten the lower clamp, and make an exact setting by using the tangent screw. The transit is then oriented for measuring magnetic azimuths. The procedure for carrying azimuths through the remaining lines in the traverse is the same as that previously described.

8-8 Azimuth Traverse

An azimuth traverse is a continuous series of lines of sight related to one another by measured angles only. The distances between the instrument stations are not measured. An azimuth traverse serves one of two purposes. The first purpose is to permit the determination of directions far removed from a beginning azimuth without the necessity of measuring distances. As an example, consider a pair of intervisible stations, such as Ridge 1 and Ridge 2 in Fig. 8-8, situated high on a ridge, and assume that the azimuth of the line joining these stations is known. A traverse is to be run in an adjacent valley, and the basis of azimuths of this traverse is to be the same as that for the line along the ridge.

To carry the azimuth down into the valley, one end of the known line, Ridge 1, is occupied and the other end of the line, Ridge 2, is used as a backsight for measuring an angle to the right, a deflection angle, or, if the traverse is to close by occupying Ridge 2, an interior angle. All the other points are occupied by the

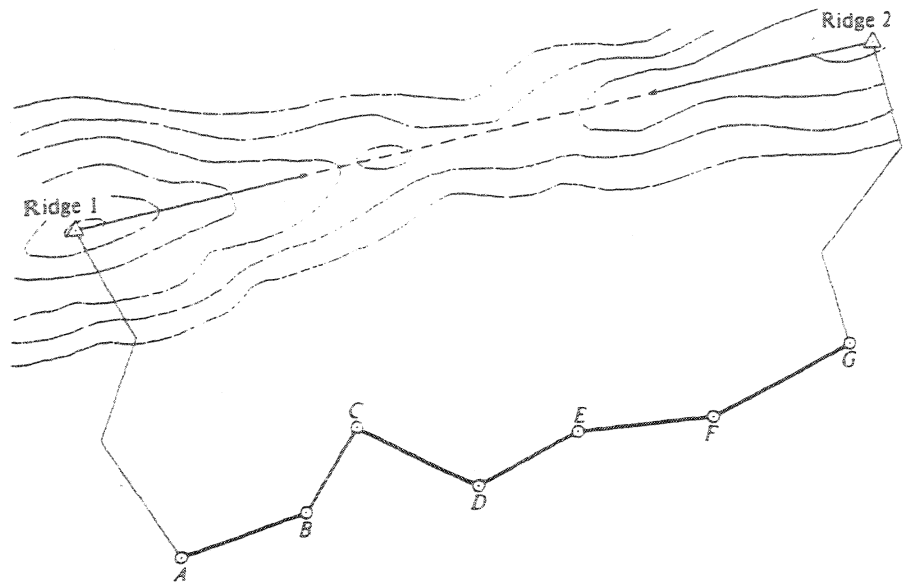


FIGURE 8-8 Azimuth traverse.

instrument, and angles are measured. That portion of the survey from Ridge 1 to *A* and the portion from *G* to Ridge 2 are azimuth traverses. The lengths of the lines from *AB* through *FG* are measured along with the angles. The angular closure can be computed, and the traverse from *A* to *G* will be on the same basis of azimuths as is the straight line from Ridge 1 to Ridge 2.

The second purpose of an azimuth traverse is to avoid carrying azimuths through extremely short traverse sides. Since the angular error will increase as the lines of sight become shorter, the desirable traverse is one with long sights. Such sights may not be practical, however, for several reasons. The ground over which the traverse must follow may be rough and the sides may have to be short to alleviate difficult taping; the traverse may run through a city where every street corner or every point at a break in the street center line is a transit station; the traverse may be run along a curving right of way where intervening brush would interfere with the measurement of the lengths of long traverse sides. In these situations the angular errors accumulating from short lines of sight can be isolated by employing an azimuth traverse of one or more stations through which the azimuth is computed. This use of an azimuth traverse is illustrated in Fig. 8-9. The azimuth is carried through lines *MN, NO, OP, PQ, QR,* and *RS,* although the azimuths of the lines forming the loops that have been cut off must also be determined. The lengths of the cutoff lines *OP* and *PQ* would not be measured. All the remaining distances would be measured.

The angular closure along the route *MNOPQRS* is adjusted by methods discussed in the previous sections. This adjustment fixes the directions of the lines *OP* and *PQ*. The loop *O, O*-1, *O*-2, *O*-3, *O*-4, *P* is then adjusted to the azimuth of *OP*. The loop *P, P*-1, *P*-2, *P*-3, *P*-4, *Q* is then adjusted to the azimuth of *PQ*.

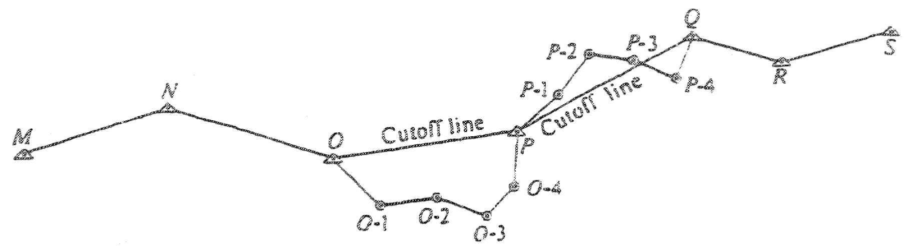


FIGURE 8-9 Azimuth traverse by cutoff lines.

8-9 Referencing a Traverse Station

As an aid in relocating a point that may become hidden by vegetation or buried beneath the surface of the ground, or as a means of replacing a point that may have been destroyed, measurements are made to nearby permanent or semipermanent objects. This process is known as *referencing* or *witnessing* the point. Property corners and, on important surveys, all instrument stations are usually referenced.

Figure 8-10 shows the locations of the witness points with respect to station *A*. If the stake at *A* cannot be found at a later time, its approximate position can be determined by locating the intersection of arcs struck with the trees as centers. The point would, of course, be determined by two arcs, but a third measurement is taken to serve as a check. If it is likely that any of the witness points will be destroyed, additional witnesses are located.

The method of recording the witnesses is the same as that used in notes for all United States land surveys (see Chapter 18). First the object is described. If it is a tree, its diameter is given. Next the bearing from the station to the witness point is given. Last the distance is recorded. If the measurement is made to any definite

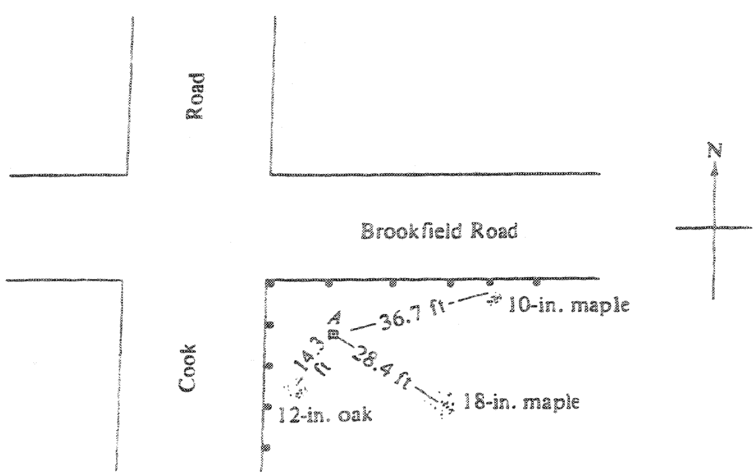


FIGURE 8-10 Referencing a point.

point, such as a nail driven in the root or side of a tree, that fact should be stated. To be of the most value in replacing a missing station, the witnesses should be less than 100 ft from the station and, if possible, the arcs should intersect approximately at right angles.

On many surveys no permanent objects may be available as witnesses. In such cases additional stakes can be driven. The method illustrated in Fig. 6-15(*f*) is the most satisfactory way of referencing a point so that it can be replaced in its original position. In this figure point P is at the intersection of lines AB and CD. If the distances from P to the four stakes are measured carefully, the point can be replaced if any two of the stakes remain, and the relocation can be checked if three of them can be found. This method is commonly used in referencing transit stations on route surveys, where it is known that all centerline stakes will be destroyed as soon as grading operations are begun.

In addition to the specific witnesses, a general description of the location of the traverse station should be given, so that a person searching for it will have a fairly good idea of where to begin searching.

8-10 Traverse Computations

The result of the field work in executing a traverse of any kind is a series of connected lines whose directions and lengths are known. The angular closure is distributed to give a series of preliminary adjusted azimuths or bearings. Errors in the measured lengths of the traverse sides, however, will tend to alter the shape of the traverse. The steps involved in adjusting a traverse whose preliminary adjusted azimuths have been determined are as follows:

1. Determine the distance that each line of the traverse extends in a north or south direction, and the distance that each line extends in an east or west direction. These distances are called, respectively, *latitudes* and *departures*.
2. Determine the algebraic sum of the latitudes and the algebraic sum of the departures, and compare them with the fixed latitude and departure of a straight line from the origin to the closing point. This presumes a closed traverse.
3. Adjust the discrepancy found in step 2 by apportioning the closure in latitudes and the closure in departures on a reasonable and logical basis.
4. Determine the adjusted position of each traverse station with respect to some origin. This position is defined by its Y coordinate and its X coordinate with respect to a plane rectangular coordinate system, the origin being the intersection of the Y axis and the X axis with the Y axis being in the direction of the meridian.

The above sequence of computations can be altered to one in which the plane rectangular coordinates are computed after step 1, and then the coordinates of the last point of the traverse are compared with the fixed coordinates of this point. This comparison establishes the closure error that is used to adjust the computed coordinates. Both of these sequences are presented in the sections that follow.

When the computations have been performed, the position of each traverse station is known with respect to any other traverse station. Furthermore each station is related in position to any other point that is defined on the same coordinate system, even though the point is not included in the traverse.

Surveying computations are performed by handheld or desk calculators, or by electronic computers for which a variety of programs are readily available. The built-in microprocessor of a total station instrument and some field data collectors are capable of solving many surveying computations (see Section 8-20).

8-11 Latitudes and Departures

The latitude of a line is the distance the line extends in a north or south direction. A line running in a northerly direction has a plus latitude; one running in a southerly direction has a minus latitude.

The departure of a line is the distance the line extends in an east or west direction. A departure to the east is considered plus; a departure to the west is minus.

In Fig. 8-11 the bearing of the line *MP* is N 23° 34′ 20″ E and its length is 791.28 ft. The bearing of *PR* is S 51° 05′ 10″ E and its length is 604.30 ft. The line *MP* has a latitude of +725.25 ft and a departure of +316.44 ft. The line *PR* has a latitude of −379.60 ft and a departure of +470.20 ft. From Fig. 8-11 it is seen that the latitude of each line is the length of the line times the cosine of the bearing angle, and that the departure of each line is the length of the line times the sine of the bearing angle. If *D* represents the length of the line and *B* is the bearing angle, then

$$\text{latitude} = D \cos B \tag{8-1}$$

$$\text{departure} = D \sin B \tag{8-2}$$

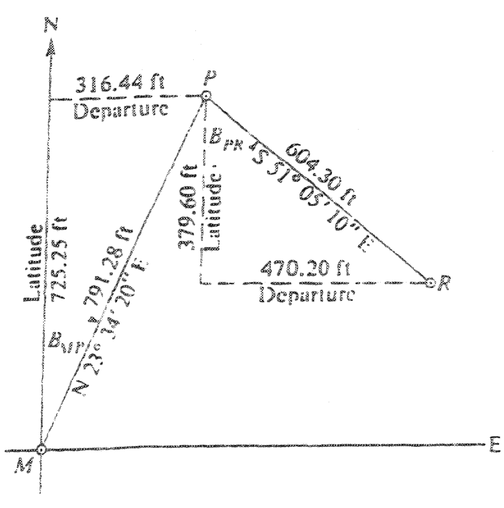


FIGURE 8-11 Latitudes and departures.

10 The Global Positioning System

10-1 Introduction

The navigation, positioning, and timing system known as the Global Positioning System (GPS) is a combination of several satellite navigation systems and concepts developed by or for the Department of Defense (DOD). The predecessors to GPS include the following satellite systems: 1) Transit, a navigation system developed by the Johns Hopkins Applied Physics Laboratory in the late 1950s and 1960s; 2) Timation, an experimental program developed for the Navy by the Naval Research Laboratory that demonstrated the ability to operate atomic clocks on board orbiting satellites and was used as a system concept for GPS; and 3) Project 621 B, an Air Force study program originated in 1964 by Aerospace Corporation and the Air Force's Space and Missile Organization. In 1968 a DOD Four-Service Executive Steering Group was established to investigate the development of a Defense Navigation Satellite System that would satisfy all of the DOD's satellite navigation requirements.

By 1972 the best characteristics of these programs had been determined to form the general system characteristics and initial design parameters for the system now known as the NAVSTAR Global Positioning System. By the late 1970s it was clear that both plane and geodetic surveying would be changed forever by this powerful space age technology. The surveying community is credited with discovering how to use the GPS signals for precise land measurements. The authors assert that no other technology has affected the surveying profession as profoundly as GPS. A number of excellent textbooks on GPS are referenced at the end of the chapter. The purpose of this chapter is to introduce the beginning surveying student to the principles of GPS so that the student can understand the limitations of the technology, attest to its accuracy and availability, and, of course, use it in various applications.

The technical and operational characteristics of GPS are organized into three distinct but somewhat arbitrary segments: the space segment, the operational control segment, and the user equipment segment. The GPS signals, which are broadcast by each satellite and carry data to both the user's equipment and ground control facilities, link the segments together into one system. Fig. 10-1 illustrates the segments of the GPS system and lists the transmitted signals. Fig. 10-2 shows the GPS satellite.

Each GPS satellite transmits signals centered on two microwave radio frequencies: 1575.42 megahertz, referred to as Link 1, or simply L1; and 1227.60 megahertz, referred to as L2. These channels lie in a band of frequencies known as the L-band, which starts just above the frequencies used by cellular telephones. The International Telecommunications Union, the radio regulation arm of the United Nations, has set aside special subbands within the L-band for satellite-based positioning systems. The L1 and L2 frequencies lie within these bands.

Such high frequencies are used for several reasons. The signals consist of a number of components. A bandwidth of about 20 MHz is required to transmit

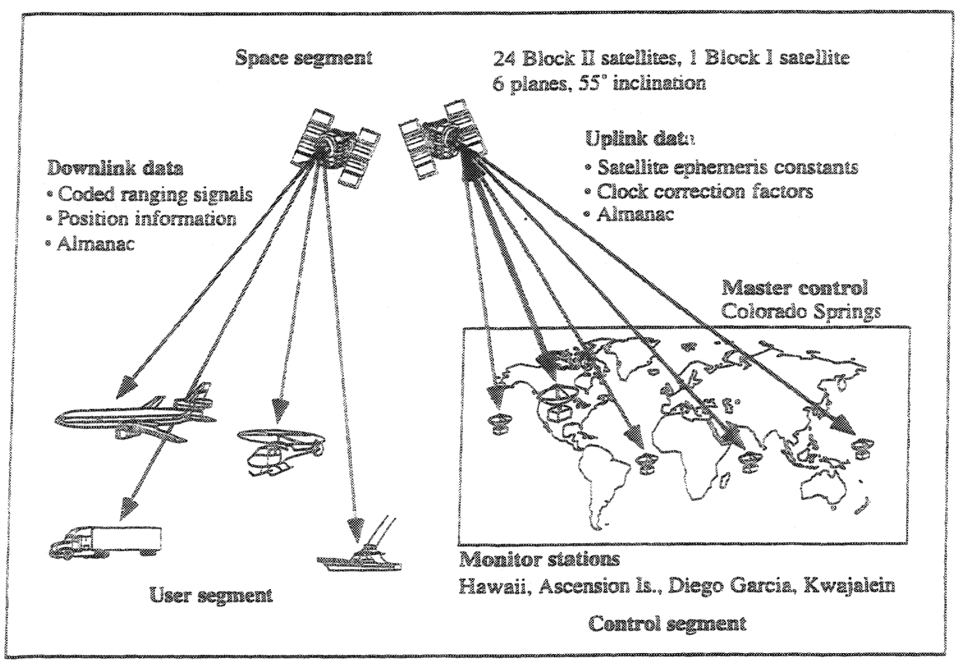


FIGURE 10-1 Segments of GPS system.

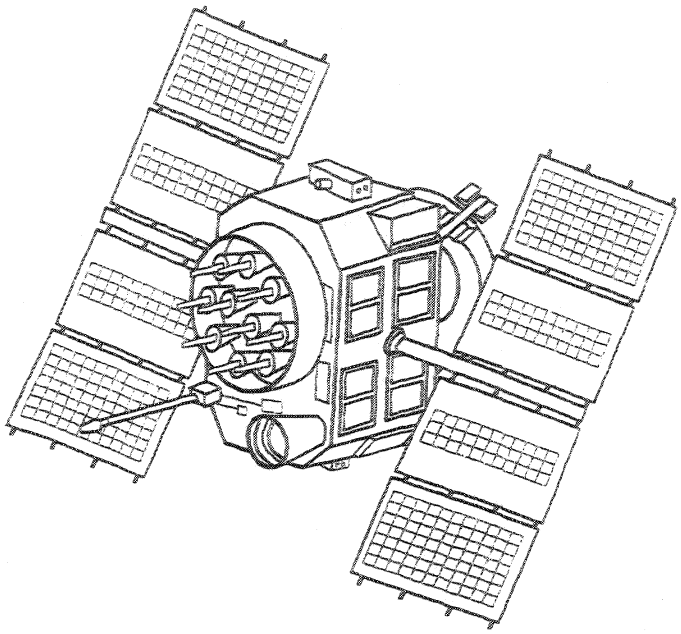


FIGURE 10-2 GPS satellite.

these components. A high, relatively uncluttered part of the radio spectrum is required for GPS-type signals.

GPS signals must provide a means for determining not only high-accuracy positions in real time but also velocities. Velocities are determined by measuring the slight shift in the frequency of the received signals due to the Doppler effect—essentially the same phenomenon associated with sound waves that gives rise to the change in pitch of a locomotive's whistle as a train passes in front of an observer at a level crossing. To measure velocities with centimeter-per-second accuracies, centimeter wavelength (microwave) signals are required.

Another reason for requiring such high frequencies is to reduce the effect of the ionosphere. The ionosphere affects the propagation speed of radio signals. The range between a satellite and a receiver derived by measuring travel times of the signal will therefore contain errors. The size of the errors gets smaller as higher frequencies are used. But even at the L1 frequency the error can amount to 30 meters for a signal arriving from directly overhead. For some GPS applications, an error of this size is tolerable. However, other applications require much higher accuracies. This is why GPS satellites transmit on two frequencies. If measurements made simultaneously on two well-spaced frequencies are combined, almost all of the ionosphere's effect can be removed.

Although high frequencies are desirable for the reasons just given, they should not be too high. For a given transmitter power, a received satellite signal becomes weaker as the frequency used becomes higher. The L-band frequencies used by GPS are therefore a good compromise between this so-called space loss and the perturbing effect of the ionosphere.

GPS signals, like most radio signals, start out in the satellites as pure sinusoidal waves or carriers. But pure sinusoids cannot be readily used to determine positions in real time. Although the phase of a particular cycle of a carrier wave can be measured very accurately, each cycle in the wave looks like the next one, so it is difficult to know exactly how many cycles lie between the satellite and the receiver. This *integer ambiguity* can be resolved using the differential techniques pioneered by surveyors and discussed in Section 10-9.

10-2 The Codes

For a user to obtain positions independently in real time, the signals must be altered in such a way that time-delay measurements can be made. This is achieved by modulating the carriers with pseudo random noise (PRN) codes.

The PRN codes consist of sequences of binary values (zeros and ones) that, at first sight appear to have been randomly chosen. But a truly random sequence can only arise from unpredictable causes over which, of course, there would be no control and which could not be duplicated. However, using a mathematical algorithm or special hardware devices called feedback shift registers, sequences can be generated that do not repeat until after some chosen interval of time. Such sequences are termed pseudo-random. The apparent randomness of these sequences makes them indistinguishable from certain kinds of noise such as the hiss heard when a radio is tuned between stations or the "snow" seen on a television screen. Although noise in a communications device is generally unwanted, in this case the noise is very beneficial.

Exactly the same code sequences are independently replicated in a GPS receiver. By aligning the replicated sequence with the received one and knowing the instant of time the signal was transmitted by the satellite, the travel time, and hence the range, can be computed. Each satellite generates its own unique codes, so a GPS receiver can easily identify which signal is coming from which satellite, even when signals from several satellites arrive at its antenna simultaneously.

Two different PRN codes are transmitted by each satellite: The C/A-code, or coarse/acquisition code, and the P-code, or precision code. The C/A-code is a sequence of 1023 binary digits, or chips, that is repeated every millisecond. This means that the chips are generated at a rate of 1.023 million per second and that one chip has a duration of about one microsecond. Each chip, riding on the carrier wave, travels through space at the speed of electromagnetic waves. A unit of distance can therefore be obtained by multiplying the time interval by this speed. One microsecond translates to approximately 300 meters. This is the wavelength of the C/A-code. Because the C/A-code is repeated every millisecond, a GPS receiver can quickly lock onto the signal and begin matching the received code with the one generated by the receiver.

The precision of a range measurement is determined in part by the wavelength of the chips in the PRN code. Higher precision can be obtained with shorter wavelengths. To get higher precision than those afforded by the C/A-code, GPS satellites also transmit the P-code. The wavelength of the P-code chips is only about 30 m, one-tenth the wavelength of the C/A-code chips: The rate at which the chips are generated is correspondingly ten times as fast, or 10.23 million per second.

The P-code is an extremely long sequence. The pattern of chips does not repeat until slightly more than 266 days, or about 2.35×10^{14} chips. Each satellite is assigned a unique seven-day segment of this code, which is initialized at midnight on Saturday each week.

The GPS PRN codes have additional useful properties. When a receiver is processing the signals from one satellite, it is essential that the signals received simultaneously from other satellites do not interfere. The GPS PRN codes have been specially chosen to be resistant to such interference. Also, the use of PRN codes results in a signal that is essentially impervious to unintentional or deliberate jamming from other radio signals, a possibility that the U.S. Department of Defense, the owner of the system, has to worry about.

The C/A-code is modulated onto the L1 carrier, whereas the P-code is modulated on both L1 and L2. This means that only users with dual-frequency GPS receivers can correct the measured ranges for the effect of the ionosphere. Users of single-frequency receivers must resort to models of the ionosphere that account for only a portion of the effect. Access to the lower-accuracy C/A-code is provided in the GPS *Standard Positioning Service* (SPS), the level of service authorized for civilian users. The *Precise Position Service* (PPS), which provides access to both the C/A-code and the P-code, is designed primarily for military uses. The SPS incorporates a further intentional degradation of accuracy, called Selective Availability (SA), which will be discussed later.

To convert the measured ranges between the receiver and the satellites to a position, the receiver must know where the satellites are. To do this in real time requires that the satellites broadcast this information. Accordingly, there is a message superimposed on both the L1 and L2 carriers along with the PRN codes. Each satellite broadcasts its own message, which consists of orbital information (the ephemeris) to be used in the position computation, the offset of its clock from GPS system time, and information on the health of the satellite and the expected accuracy of the range measurements.

The message also contains almanac data from the other satellites in the GPS constellation, as well as their health status and other information. The almanac data, a crude description of the satellite orbit, are used by the receiver to determine the location of each satellite. The receiver uses this information to acquire the signals quickly from satellites that are above the horizon but are not yet being tracked. So, once one satellite is tracked and its message decoded, acquisition of the signal from other satellites is quite rapid.

The broadcast message also contains another very important piece of information for receivers that track the P-code. As mentioned earlier, the P-code segment assigned to each satellite is seven days long. A GPS receiver with an

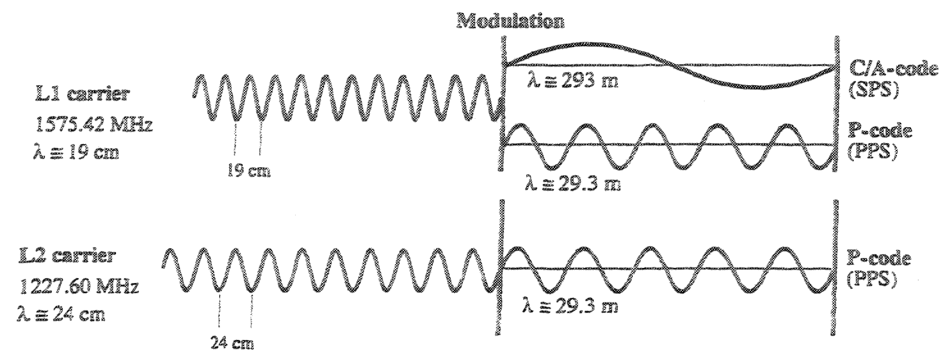


FIGURE 10-3 GPS satellite signals.

initially unsynchronized clock has to search through its generated P-code sequence to try to match the incoming signal. It would take many hours to search through just one second of the code, so the receiver needs some help. It gets this help from a special word in the message called the Hand-Over Word (HOW), which tells the receiver where in the P-code to start searching. Fig. 10-3 describes the signals.

10-3 Satellite Positioning

The GPS satellites provide the user with the capability of determining his position, expressed in Cartesian coordinates X_r, Y_r, Z_r or latitude, longitude, and height. Fundamentally this is accomplished using the principles of resection by measuring ranges. Think of the GPS satellites as frozen in space at a specific instant. The space coordinates X_s, Y_s, and Z_s, relative to the center of mass of the earth of each satellite, are known because the ephemeris (the orbit) of the satellites is determined by the operational control segment of the GPS system and broadcast to the users by each of the satellites.

If the ground receiver employed a clock that was set precisely to the GPS system time, the true distance (range) to each satellite could be accurately measured by recording the time required for the satellite signal to reach the receiver. Using this technique, ranges to only three satellites would be needed since the intersection of three spheres whose radii are the respective ranges yields the three unknowns X_r, Y_r, and Z_r and could be determined from three range equations. Fig. 10-4 illustrates the problem, and Eq. (10-1) expresses the solution.

$$R_j = \left[(X_{sj}-X_r)^2+(Y_{sj}-Y_r)^2+(Z_{sj}-Z_r)^2\right]^{\frac{1}{2}},\ j \geq 3 \qquad (10\text{-}1)$$

Unfortunately there cannot be perfect synchronization between the satellite clocks and the receiver clock, and this clock offset or error introduces an unacceptable positional error. Moreover, as discussed later, both clocks are different

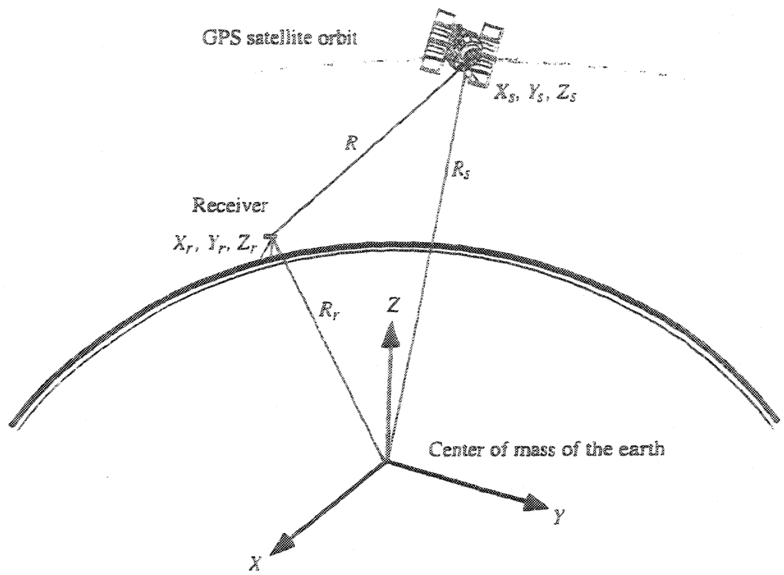


FIGURE 10-4 *Range from satellite to ground receiver.*

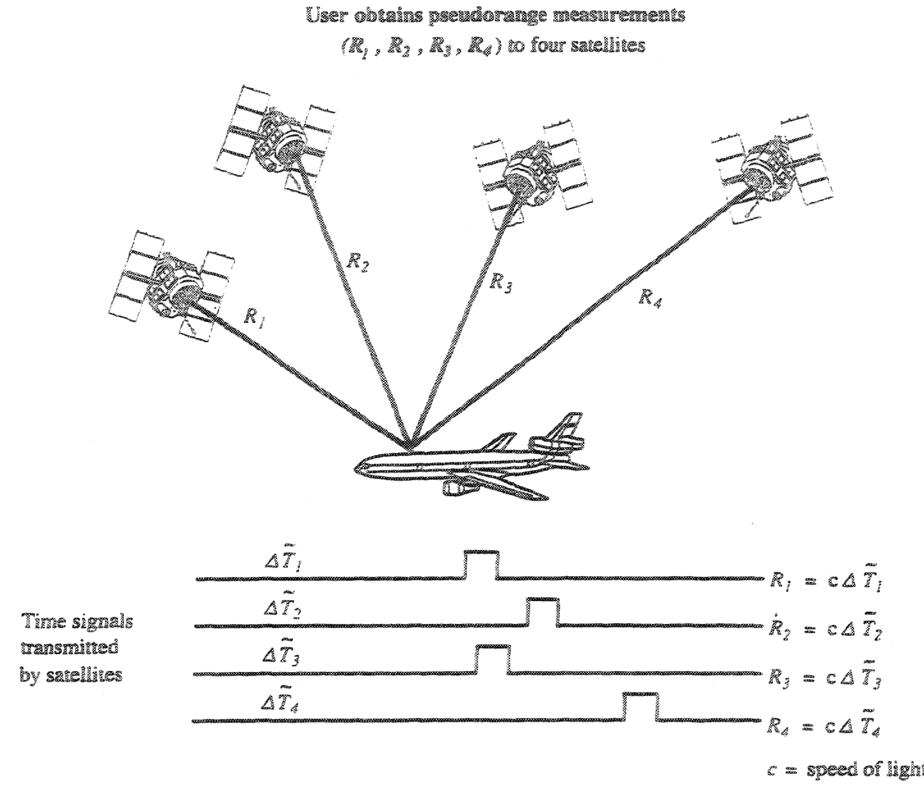
than the GPS system time because of the usual errors (such as drift) associated with any timing device. The receiver clock problem can be circumvented by measuring simultaneously to at least four satellites and solving for this unknown clock error. These measured distances are called pseudoranges since they are the true ranges affected by the satellite and receiver clock error. In the case where only the satellite and receiver clock errors are considered, the mathematical model for the pseudoranges has the following form:

$$R_{rj} = \rho_{rj} + c(\Delta T_r - \Delta T_j), \; j \geq 4 \tag{10-2}$$

The subscript j refers to the number of satellites. The terms R_j, ρ_j, ΔT_r, ΔT_j are the measured pseudorange, the geometric range, the receiver and the satellite clock error, respectively, and c is the speed of light. The geometric distance ρ_j is, in Cartesian coordinates,

$$\rho_{rj} = [(X_j - X_r)^2 + (Y_j - Y_r)^2 + (Z_j - Z_r)^2]^{\frac{1}{2}} \tag{10-3}$$

The satellite clock error, ΔT_j, can be considered to be known from the ephemeris message or combined with ΔT_r and solved for as part of the solution depending on accuracy requirements and mode of operation. If there are four satellites and ΔT_j is known, a unique solution for X_r, Y_r, Z_r, and ΔT_r results, Fig. 10-5. This result can be easily converted to latitude, longitude and height with the appropriate equations.



Pseudoranges	Position Equations
$R_1 = c\Delta\tilde{T}_1$	$(X_1 - X_r)^2 + (Y_1 - Y_r)^2 + (Z_1 - Z_r)^2 = (R_1 - c\Delta T_r)^2$
$R_2 = c\Delta\tilde{T}_2$	$(X_2 - X_r)^2 + (Y_2 - Y_r)^2 + (Z_2 - Z_r)^2 = (R_2 - c\Delta T_r)^2$
$R_3 = c\Delta\tilde{T}_3$	$(X_3 - X_r)^2 + (Y_3 - Y_r)^2 + (Z_3 - Z_r)^2 = (R_3 - c\Delta T_r)^2$
$R_4 = c\Delta\tilde{T}_4$	$(X_4 - X_r)^2 + (Y_4 - Y_r)^2 + (Z_4 - Z_r)^2 = (R_4 - c\Delta T_r)^2$

R_i = Pseudorange (i = 1, 2, 3, 4)

Pseudorange includes actual distance between satellite and user plus satellite clock bias, user clock bias, atmospheric delays, and receiver noise.

Satellite clock bias and atmospheric delays are compensated for by incorporation of deterministic corrections prior to inclusion into the solution.

X_i, Y_i, Z_i = satellite position (i = 1, 2, 3, 4)

Satellite position broadcast in navigation 50-Hz message

Receiver solves for:

X_r, Y_r, Z_r = receiver position

ΔT_r = user clock bias

FIGURE 10-5 Receiver position solution using signals from four satellites.

For many applications this simple "navigation" solution will suffice. These applications deal with facilities management (FM in the literature), which is the process of creating a spatially referenced inventory of land information features, such as light poles, stop signs, on-off ramps, and so on. It is critical that the modern surveyor understand the accuracy and limitation of this type of positioning, as it will become more important in the future as the GPS accuracy improves as a result of important future enhancements.

In considering the error sources in a single-point positioning from pseudoranges, it is first necessary to discuss two errors that are deliberately introduced by the DOD. Selective Availability (SA) is the deliberate degradation in the GPS satellite orbit information (the navigation message) and timing by the DOD. The total error introduced today (1996) is limited by policy to $[\sigma_x^2 + \sigma_y^2]^{\frac{1}{2}} \leq 50$ m. The other error introduced by the DOD is Anti-Spoofing (A-S), which is the encryption process used to deny unauthorized access to a modification of the P-code known as the Y-code. The A-S process prevents potential enemies from spoofing or fooling a friendly GPS user of the Y-code.

Other errors in the pseudoranges are listed in Table 10-1 and are briefly discussed below.

The radio waves emitted by the satellites pass through the earth's ionosphere and troposphere and create a delay in the arrival time of the signals and hence an error in the range. Errors exist in the clocks in the satellites, the ephemeris or orbit of the satellites, in the receiver whose position is to be determined, and because of a phenomenon called multipath. Multipath is caused by objects near the receiver antenna that reflect the satellite signals to the antenna after hitting the object. Those signals travel a longer path than those traveling directly to the antenna and hence cause an error. This is a condition that can largely be avoided by the surveyor in the field by placing the receiver antenna in a place free from reflected signals (for example, away from a building wall). Figure 10-6 shows a GPS receiver and antenna.

Another important aspect of GPS positioning with a single receiver is that in the future it is highly likely that the accuracy of such positioning will improve. Many experts believe that the instantaneous "navigation" accuracy of GPS will

TABLE 10-1 Errors in Single-Point Positioning

Error Source	Range Error (σ) in Meters
Selective availability (SA)	24.0
Atmospheric delay	
Ionospheric	7.0
Tropospheric	0.7
Clock and ephemeris error	3.6
Receiver noise	1.5
Multipath	1.2
Total range error	25.3
Resulting horizontal accuracy $\sqrt{\sigma_x^2 + \sigma_y^2}$	50.0

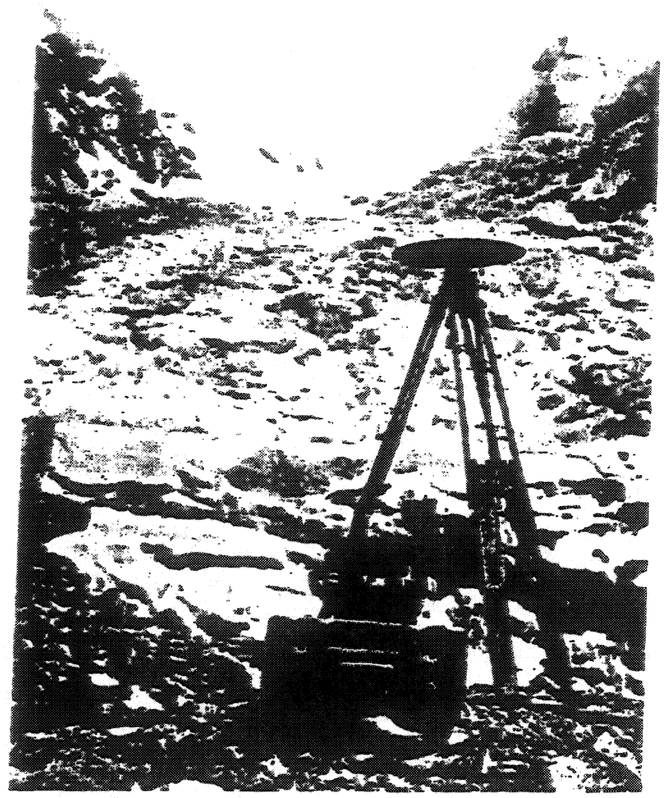


FIGURE 10-6 GPS receiver and antenna. Courtesy of Trimble.

improve to a level of 2–5 meters by the year 2005. This accuracy level will not satisfy the requirements for boundary or property surveying but will, as stated earlier, satisfy many requirements associated with facilities management and other tasks.

10-4 Differential GPS

By far the most significant application of GPS for surveyors is to accomplish highly accurate measurement of a vector (as in mathematics, both magnitude and direction) between two points, that is, ΔX, ΔY, and ΔZ. Usually one point is known with respect to the appropriate geodetic datum, and one or more additional points are determined with respect to the known point. In this fashion most of the errors listed in Table 10-1 tend to cancel out. For example, SA is nearly eliminated by differencing simultaneous measurements at two stations. This is because SA and other errors are common to both of the receivers, that is, the receiver occupying the known point and the receiver situated at the point to be determined.

The actual equations used to reduce GPS observations today are more complicated than those provided below. However, the basic principles can be grasped from the following discussion.

In Fig. 10-7, if we are only concerned with $X_2 - X_1$, $Y_2 - Y_1$, and $Z_2 - Z_1$, we can write equations differencing the pseudoranges to satellites A, B, C, and D from stations 1 and 2 as

$$R_{2A} - R_{1A} = (\rho_{2A} - \rho_{1A}) + c(\Delta T_2 - \Delta T_A) - c(\Delta T_1 - \Delta T_A) \tag{10-4}$$

or

$$R_{2A} - R_{1A} = (\rho_{2A} - \rho_{1A}) + c(\Delta T_2 - \Delta T_1) \tag{10-5}$$

which shows how the satellite clock error cancels out. Similarly

$$R_{2B} - R_{1B} = (\rho_{2B} - \rho_{1B}) + c(\Delta T_2 - \Delta T_1) \tag{10-6}$$

$$R_{2C} - R_{1C} = (\rho_{2C} - \rho_{1C}) + c(\Delta T_2 - \Delta T_1) \tag{10-7}$$

$$R_{2D} - R_{1D} = (\rho_{2D} - \rho_{1D}) + c(\Delta T_2 - \Delta T_1) \tag{10-8}$$

where ρ_{rj} is given by Eq. 10-3, with $r = 1, 2$ and $j = A, B, C, D$. Since R_{1j} and R_{2j} are computed from the measurements at the two stations; and in this simplified example c, the speed of electromagnetic waves, is considered known; and the satellite coordinates X_j, Y_j, Z_j, $j = A, B, C, D, \ldots$; and the fixed-station coordinates, say X_1, Y_1, and Z_1, are considered known, the above four equations can be solved

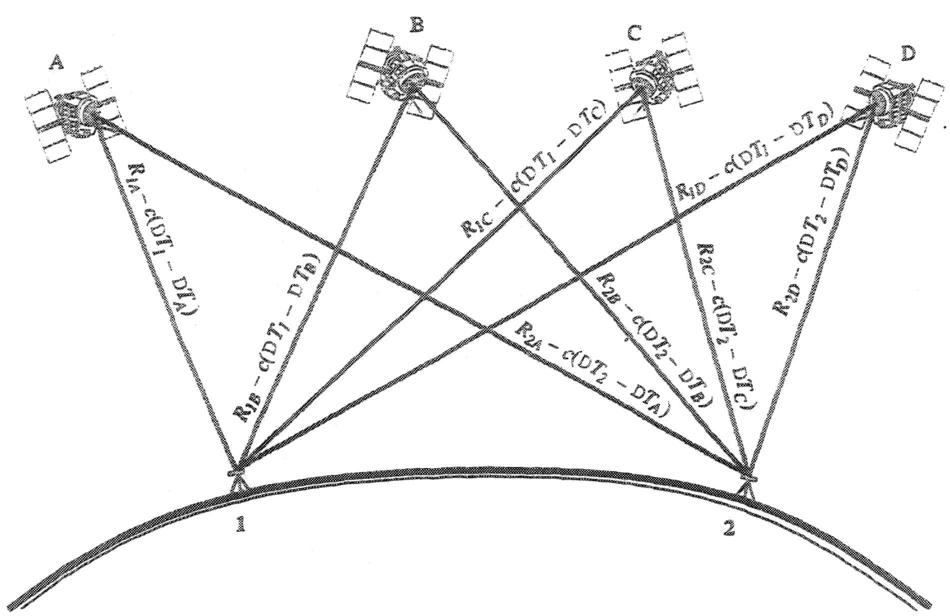


FIGURE 10-7 Satellite and ground receiver configuration for differential GPS.

for the four unknowns $(\Delta T_2 - \Delta T_1)$, X_2, Y_2, Z_2. The above technique is called single difference differential pseudoranging. This technique is common today and can be used to obtain accuracies of $\sigma = \pm 1$ m over distances of up to 50 km. In practice, of course, many more observations are taken on as many satellites as possible, resulting in the need for a least-squares solution to the problem.

Today's instruments are capable of making pseudorange measurements using either the C/A- or the P(Y)-code transition modulations. Computations of the precise vector between two points when the differential carrier phase measurement is used is, of course, quite similar to differential pseudoranging. Instead of differencing the C/A-code pseudoranges, a measurement comprised of a large number of whole (integer) carrier cycles plus a fractional cycle measurement is differenced. The integer portion of this measurement is unknown and must be estimated and ultimately solved for; this measurement can be thought of as an ambiguous pseudorange. Rewriting the right side of Eqs. 10-5 to 10-8 yields the following:

$$R_{2A} - R_{1A} = \Delta\Phi_{21A} - N_{21A} \qquad (10\text{-}9)$$

$$R_{2B} - R_{1B} = \Delta\Phi_{21B} - N_{21B} \qquad (10\text{-}10)$$

$$R_{2C} - R_{1C} = \Delta\Phi_{21C} - N_{21C} \qquad (10\text{-}11)$$

$$R_{2D} - R_{1D} = \Delta\Phi_{21D} - N_{21D} \qquad (10\text{-}12)$$

or

$$R_{2j} - R_{1j} = \Delta\Phi_{21j} - N_{21j} \quad ; \; j = A, B, C, D, \ldots \qquad (10\text{-}13)$$

where

$$R_{2j} - R_{1j} = (\rho_{2j} - \rho_{1j}) + c(\Delta T_2 - \Delta T_1) \qquad (10\text{-}14)$$

$$\Delta\Phi_{21j} = \Phi_{2j} - \Phi_{1j} \qquad (10\text{-}15)$$

$$N_{21j} = N_{2j} - N_{1j} \qquad (10\text{-}16)$$

and

$$N_{rj} = (n_c \lambda_c) \qquad r = 1, 2 \qquad (10\text{-}17)$$

$$\Phi_{rj} = \left(\frac{\theta}{2\pi}\lambda_c\right)_{rj} \qquad j = A, B, C, D, \ldots \qquad (10\text{-}18)$$

and

n_c = integral number of carrier wavelengths
λ_c = carrier wavelengths
θ = phase angle of carrier

Comparing Eqs. 10-9 to 10-12 and Eq. 10-14, it follows that the left side of Eqs. 10-9 to 10-12 contain three unknown coordinates (coordinates of station 2), four unknown biases (N_{21j}, $j = A, B, C, D$) and one unknown clock term $(\Delta T_2 - \Delta T_1)$, giving a total of eight unknowns.

These equations, called single-difference equations, are solved for by observing ranges at numerous epochs and the unknowns solved for by using least squares.

10-5 Kinematic GPS

With the advent of dual-frequency GPS receivers and sophisticated and clever techniques like those developed by NGS (see references), it is possible using differential GPS to obtain extremely high accuracies in real time while a rover receiver is moving. The trick is to be able to determine which wavelength has arrived at the rover. This is generally referred to as real-time ambiguity resolution on-the-fly (OTF). A number of sophisticated algorithms have been developed to do this, and it is generally possible to determine the correct wavelength (referred to as getting back "on lock") within seconds with only five satellites in view while the rover is moving.

The real-time technique would be common in current GPS surveying if it didn't cost the surveyor significantly more than post-processed results. As the cost of two-wavelength receivers comes down, it will become the practice of choice. Dramatic price reductions for all GPS receivers have occurred over the past few years.

The ambiguity resolution equations referred to in this section are beyond the scope of this textbook, however, the principles of satellite positioning outlined in this chapter are valid no matter what GPS method is used.

10-6 Accuracy of Differential Techniques

The errors shown in Table 10-1 take on a form similar to that of electronic distance measuring devices, that is,

$$\sigma_{GPS} = [\sigma_A^2 + \sigma_B^2(d)]^{\frac{1}{2}} \qquad (10\text{-}19)$$

where σ_{GPS} can be thought of as the standard error of a distance determined by GPS; σ_A is a constant error, usually 5 mm $\le \sigma_A \le$ 15 mm for high-quality two-wavelength devices; and σ_B is a distance (d) dependent error that ranges from $d \times \frac{1}{10^{[\text{illegible}]}}$ m to $d \times \frac{1}{10^{[\text{illegible}]}}$ m depending on the type of receiver used and the error model in postprocessing. Therefore, for medium length (10 km) distances, an average error of the distance between the base and rover station might be $\sigma_{GPS} = \pm 2$ cm under good conditions. Amazingly this result is true even if the receiver is used in a static or kinematic mode. Therefore GPS can provide the same or better accuracy than the best traditional surveying techniques over distances greater than 1000 m or so.

10-7 Transformation of GPS Results

Denoting the Cartesian (rectangular) coordinates of a point in space by X, Y, and Z, and assuming an ellipsoid of revolution with the same origin as the Cartesian coordinate system, the point can also be expressed by the ellipsoidal coordinates φ, λ, and h, as seen in Fig. 10-8. The relation between the Cartesian coordinates and the ellipsoidal coordinates is

$$X = (R_N + h) \cos \varphi \cos \lambda \tag{10-20a}$$

$$Y = (R_N + h) \cos \varphi \sin \lambda \tag{10-20b}$$

$$Z = \left(\frac{b^2}{a^2} R_N + h\right) \sin \varphi \tag{10-20c}$$

where R_N is the radius of curvature in the prime vertical, thus

$$R_N = \frac{a^2}{[a^2 \cos^2 \varphi + b^2 \sin^2 \varphi]^{\frac{1}{2}}} \tag{10-21}$$

and a and b are the semiaxes of the reference ellipsoid. The Cartesian coordinates are related to WGS-84, which is the datum (reference system) used by the GPS. There is little practical difference between WGS-84 and NAD 83.

Eqs. 10-20a, b, and c transform ellipsoidal coordinates φ, λ, and h into Cartesian coordinates X, Y, and Z. For GPS applications the inverse transformation is more important because the Cartesian coordinates are given and the ellipsoidal coordinates sought. Therefore the ellipsoidal coordinates φ, λ, and h must be computed from the Cartesian coordinates X, Y, and Z. Usually this problem is solved iteratively, although a solution in an explicit form is possible. This development is beyond the scope of this book and is fully explained in the references.

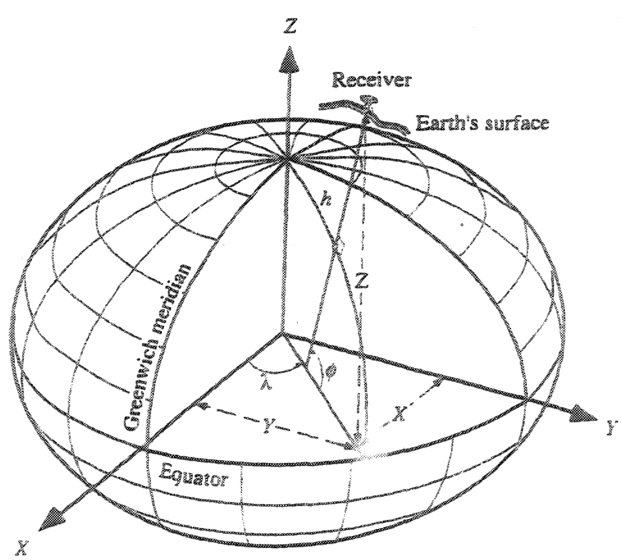


FIGURE 10-8 Coordinate reference system of GPS and conversion to latitude, longitude, and ellipsoidal heights.

10-8 Height Determination Using GPS

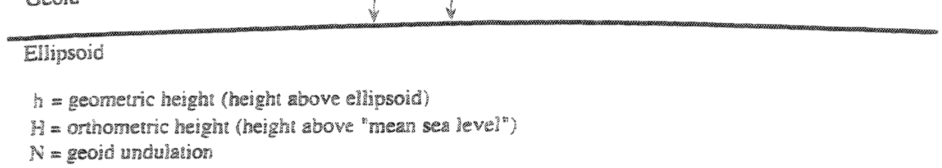
From the preceding section it is clear that the ellipsoid height h in Fig. 10-8 can be obtained using Eqs. 10-20a, b, and c, but the distance N in Fig. 10-9 must be known to determine the orthometric height H. Orthometric heights are obtained by leveling. The relationship is usually defined as

$$H = h - N$$

The X, Y, and Z coordinates determined by GPS must be transformed using the inverse of Eqs. 10-20a, b, and c but are still not useful when comparing orthometric heights that were determined by spirit leveling. The reason for this is the fact that leveling is affected by the earth's gravity field and the datum for leveling is usually an equipotential surface near or coinciding with mean sea level. This surface is called the geoid. The distance N is the separation between the ellipsoid and the geoid.

Today N has been modeled in the United States to $\sigma = \pm 10$ cm. These values $N(\varphi, \lambda)$ are available from the NGS. Moreover since the relative accuracy of the modeled geoid is very good and N changes slowly over short distances, the following procedure can be used:

1. First, occupy an existing (NAVD 88) benchmark with height H_{BM} and compare N_{GPS} from $N_{GPS} = h_{GPS} - H_{BM}$ with the modeled N, denoted by N_{MDL}
2. Determine a correction to the modeled N by $\Delta N = N_{GPS} - N_{MDL}$
3. If more than one benchmark is available nearby, determine ΔN at several points and average the result.
4. Use ΔN to correct the GPS determined heights using $H = h_{GPS} - (N_{MDL} + \Delta N_{AVG})$.



h = geometric height (height above ellipsoid)
H = orthometric height (height above "mean sea level")
N = geoid undulation

H = h - N

FIGURE 10-9 Relationship between ellipsoidal height and orthometric height.

10-9 Surveying with GPS

The most significant difference between GPS and traditional surveying techniques is the fact that GPS does not require line of sight between measured points. This can be taken into account advantageously to increase the efficiency and productivity of GPS surveying. For instance, when inexpensive dual-frequency GPS receivers are available, the baseline between the measured points may be on the order of 10–100 km. As a result very few *base stations* are needed to serve a very large area. With this scenario GPS surveys can be performed using only the rover receiver. The base station data can be downloaded through the network using different on-line services. When only single-frequency GPS receivers are available, the above scenario is more difficult since the baselines should be either shorter (approximately 10–20 km) or regional ionospheric corrections should be supplied with the base station data. This, however, is not the current practice because it is more convenient for GPS surveying companies to establish their own base stations for each project.

A typical GPS survey consists of the following steps:

1. Identifying sites and their accessibility.[†]
2. Recording the geometry of obstructions for each site.
3. Planning the GPS observations.
4. Conducting the GPS observations.
5. Estimating the baseline lengths.
6. Performing network adjustment.
7. Evaluating the adjustment results.
8. Performing format conversions according to user requests.

Currently 25 GPS satellites are in orbit (1 Block I and 24 Block II). As a result, if an observing site is clear of obstructions, planning will not be required because in this case the satellite coverage is good at all times, as shown in Fig. 10-10. This figure shows the number of visible satellites over a 24-hour period. The presence of obstructions can affect satellite visibility substantially, and therefore planning is necessary to determine the periods of best satellite coverage for making the GPS observations. In general, it is best to have a clear view of the sky above about 15° elevation above the horizon.

Leapfrogging techniques are common in GPS surveying. Currently the most common technique is to use three or more receivers in stop-and-go mode. In this mode one of the receivers is placed on the base station, preferably the one with the fewest obstructions. The other receivers (rover receivers) are placed on the points to be surveyed for the time required to determine the integer ambiguities necessary for cm-level surveying. The receiver firmware tracks internal indicators (for example, the quality of the geometry formed by the satellites being tracked, and called PDOP, with time of continuous tracking) for determining the length of time required to estimate the integer ambiguities successfully. Once the integer ambiguities are determined, the rest of the points can be surveyed with just one

[†]Access by road and easy intervisibility of the azimuth mark are important factors in planning a GPS survey.

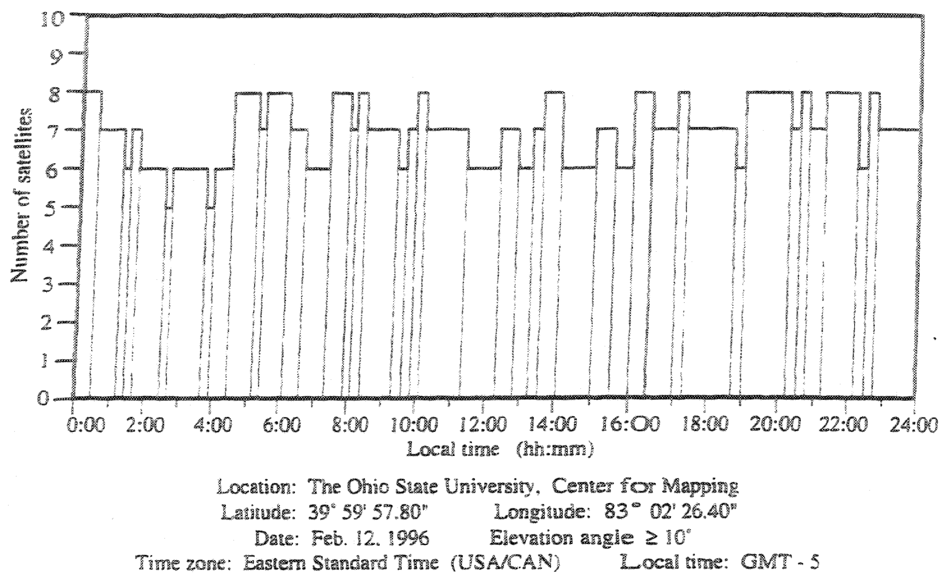


FIGURE 10-10 Satellite visibility.

epoch of observations, assuming that lock to at least four satellites with good geometry is maintained. The receivers have indicators informing the users if a certain point should be reobserved for longer periods because lock to at least four satellites was not maintained.

A real-time implementation of this system referred to as Real-Time Kinematic (RTK) will resolve the integer ambiguities and perform cm-level positioning in real time. In this mode the ambiguity resolution is referred to as OTF ambiguity resolution. The advantage of the real-time system is that it does not require any postprocessing since the positions are computed in real time. The major disadvantage, however, is that it requires a direct communication link, which makes the system more expensive and more difficult to implement. The length of time required to perform ambiguity resolution and cm-level positioning depends primarily on the type of receivers used. With dual-frequency GPS receivers, the time required to perform OTF ambiguity resolution ranges from a few seconds to 1 or 2 minutes. With single-frequency receivers this time is much longer, on the order of 2 to 10 or 15 minutes, depending on the errors affecting the carrier phase measurements. However, by starting from a known baseline or performing an antenna swap, the ambiguity resolution time ranges from few seconds to one minute.

An antenna swap is performed by occupying the base station receiver on the base station and the roving receiver on a point nearby. After a few minutes of receiving signals at both points from at least four satellites, the two antenna are interchanged from one tripod to the other, care being taken to maintain lock on all four (or more) satellites. Data are collected for a few minutes, after which the two antenna are again returned to their original tripods, with lock maintained on the satellites. This process eliminates the integer ambiguity at the base station.

TABLE 10-2 Observational Strategies for cm-level Relative Positioning

Strategy	Description	Remarks
Static	Stationary.	Long observation time; yields baselines and ambiguities simultaneously (30–120 minutes).
Kinematic	One stationary, one moving.	Integer ambiguities are determined at start with static initialization. Maintain lock to at least four satellites.
Pseudostatic	One stationary, one moving. Moving receiver revisits points at least once at hourly intervals.	Baselines and integer ambiguities are determined simultaneously.
On-the-fly	One stationary, one moving.	Integer ambiguities are determined without static initialization (ambiguity resolution).

TABLE 10-3 Dual-Frequency Receiver List

Company	Model	Signal Tracked	Max. # Satellites Tracked	Size (in) (W x H x D)	Wt. (Lb.)	Power Use (Watts)	Price ($US in 1996)
Ashtech	Z-12 Field Surveyor	L1, CA and P-code L2, P-code	12	8.5 × 3.9 × 8.0	8.8	12	32,000
Leica	Kinematic GPS Receiver	L1, CA and P-code L2, P-code	9	8.0 × 7.0 × 3.0	7.5	8	17,845
Topcon America	Turbo SII	L1, CA and P-code L2, P-code	8	8.6 × 4.1 × 1.9	2.2	5	22,950
Trimble	Site Surveyor SSi	L1, CA and P-code L2, P-code	12	9.8 × 4.0 × 11.0	6.8	9	23,000

If lock to at least four satellites cannot be maintained, re-initialization is required either starting from a known baseline or by performing an antenna swap, which makes this mode of RTK difficult to implement in practice. Observational strategies that were common at the time of this writing are shown in Table 10-2. The student may want to speculate about the future from this table.

There are a variety of receivers available on the market today. Table 10-3 presents a summary of selected two-frequency receivers. These two-frequency devices are commonly used for highly precise work, but single-frequency receivers can accomplish a large number of surveying tasks.

PROBLEMS

10-1. Why was the L band radio frequency chosen for the GPS system?

10-2. Given the range,

$$\rho(\text{m}) = 20{,}810{,}092.13300,$$

satellite coordinates (kms) $X_s = -1734.357680$ $Y_s = -16398.367040$ $Z_s = 21091.709751$

and receiver coordinates (m) $X_r = -56703.2791$ $Y_r = -4986718.0872$

calculate the Z-coordinate of the receiver.

10-3. If a satellite clock is biased by +150 nanoseconds and the receiver clock is biased by −10 nanoseconds, what is the error in the geometric range from this source?

10-4. Assuming the range in problem 2 is incorrect by 100 m, what is its affect on the Z-coordinate of the receiver?

10-5. Compute the error in distance between the two GPS stations when the constant error, σ_A, is 0.005 m and the distance dependent error is $1/10^7$ and d = 10,000 meters.

10-6. For a latitude of 40° 00′ 00″ and a longitude of 83° 02′ 27″ W using the GRS 80 ellipsoid (see page 370), and a height h = 100 meters, compute the equivalent cartesian coordinates *X*, *Y*, and *Z*.

10-7. Given the following data H_{BM} = 1000.00 m, h_{GPS} = 1020.16 m, N_{MDL} = 20.46 m. Find the correction ΔN to apply to points in the vicinity of H_{BM}.

10-8. In problem 7, would the correction ΔN be applicable 20 kilometers from H_{BM}?

10-9. Assume that you, as a practicing surveyor, have single frequency GPS equipment. Your fastidious client wants you to determine the length of a line 700 miles long. What caveats regarding measurements would you present to your client?

10-10. Using figure 10-10, estimate the approximate hours during the 24 hour period when 8, 7, 6, and 5 satellites are visible to an observer.

10-11. If a manufacturer told you that his receiver could perform OTF ambiguity resolution in the presence of only 5 satellites, with AS turned on, in 10 seconds, with 95% reliability. Would you consider this receiver state of the art in 1998?

10-12. Aside from SA, the largest error in GPS ranges results from what factor?

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15 Topographic Surveys

15-1 Introduction

Topographic surveying is the process of determining the positions, on the earth's surface, of the natural and man-made features of a given locality and of determining the configuration of the terrain. The location of the features is referred to as *planimetry*, and the configuration of the ground is referred to as *topography* or *hypsography*. The purpose of the survey is to gather data necessary for the construction of a graphical portrayal of planimetric and topographic features. This graphical portrayal is a topographic map. Such a map shows both the horizontal distances between the features and their elevations above a given datum. On some maps the character of the vegetation is shown by means of conventional symbols.

Topographic surveying or mapping is accomplished by ground methods requiring the use of the transit or theodolite, total station, level, hand level, tape, and leveling rod in various combinations. Total station instruments are used to advantage in topographic surveying. The vast majority of topographic mapping is accomplished by aerial photogrammetric methods, as described in Chapter 16. In the photogrammetric methods, however, a certain amount of field surveying and field editing must be done by ground methods described in this chapter.

The preparation of a topographic map, including the necessary control surveys, is usually the first step in planning and designing an engineering project. Such a map is essential in the layout of an industrial plant, the location of a railway or highway, the design of an irrigation or drainage system, the development of hydroelectric power, city planning and engineering, and landscape architecture. In time of war, topographic maps are essential to persons directing military operations.

15-2 Scales and Accuracy

Since a topographic map is a representation, on a comparatively small plane area, of a portion of the surface of the earth, the distance between any two points shown on the map must have a known definite ratio to the distance between the corresponding two points on the ground. This ratio is known as the scale of the map. As stated in Section 8-34, this scale can be expressed in terms of the distance on the map, in inches, corresponding to a certain distance on the ground, in feet. For example, a scale may be expressed as 1 in. = 200 ft. The scale can be expressed also as a ratio, such as 1 : 6000, or as a fraction, such as 1/6000. In either of these two cases, 1 unit on the map corresponds to 6000 units on the ground. A fraction indicating a scale is referred to as the *representative fraction*. It gives the ratio of a unit of measurement on the map to the corresponding number of the same units on the ground.

The scale to which a map is plotted depends primarily on the purpose of the map, that is, the necessary accuracy with which distances must be measured or scaled on the map. The scale of the map must be known before the field work is begun, since the field methods to be employed are determined largely by the scale to which the map is to be drawn. When the scale is to be 1/600 (1 in. = 50 ft) distances can be plotted to the nearest $\frac{1}{2}$ or 1 ft, whereas if the scale is 1/12,000 (1 in. = 1000 ft) the plotting will be to the nearest 10 or 20 ft and the field measurements can be correspondingly less precise.

15-3 Methods of Representing Topography

Topography may be represented on a map by hachures, contour lines, form lines, or tinting. *Hachures*, or hill shading, are a series of short lines drawn in the direction of the slope. For a steep slope the lines are heavy and closely spaced. For a gentle slope they are fine and widely spaced. Hachures are used to give a general impression of the configuration of the ground, but they do not give the actual elevations of the ground surface.

A *contour line*, or *contour*, is a line that passes through points having the same elevation. It is the line formed by the intersection of a level surface with the surface of the ground. A contour is represented in nature by the shoreline of a body of still water. The *contour interval* for a series of contour lines is the constant vertical distance between adjacent contour lines. Since the contour lines on a map are drawn in their true horizontal positions with respect to the ground surface, a topographic map containing contour lines shows not only the elevations of points on the ground, but also the shapes of the various topographic features, such as hills, valleys, escarpments, and ridges.

Figure 15-1 is the classical illustration used to show the relationship between the configuration of the ground and the corresponding contour lines. This illustration used to be printed on the backs of the U.S. Geological Survey quadrangle maps along with an explanation of the topographic map and how it is interpreted and used. Unfortunately the U.S. Geological Survey discontinued this feature of its quadrangle series in the early 1950s. The upper part of the illustration shows a stream lying in a valley between a cliff on the left and a rounded hill on the right.

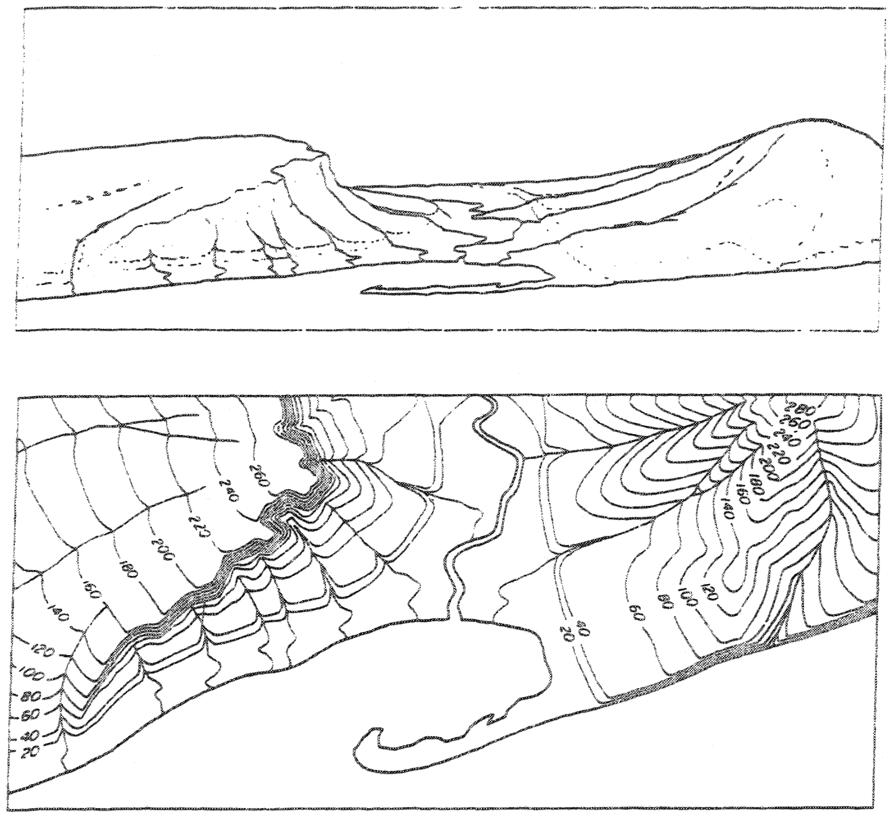

FIGURE 15-1 Contour line representation of terrain. By permission of U.S. Geological Survey.

The stream is seen to empty into the ocean in a small bay protected by a sand hook. Other features such as terraces, gullies, and a gentle slope behind the cliff can be identified. An abrupt cliff to the right of the sand hook plunges almost vertically to the ocean. The lower part of the figure is the contour line or topographic map representation of this terrain. The contour interval of this map is 20 units and could represent either 20 ft or 20 m.

On maps intended for purposes of navigation, peaks and hilltops along the coast are sometimes shown by means of *form lines.* Such lines resemble contours but are not drawn with the same degree of accuracy. All points on a form line are supposed to have the same elevation, but not enough points are actually located to conform to the standard of accuracy required for contour lines.

On aeronautical charts and on maps intended for special purposes, such as those that may accompany reports on some engineering projects, elevations may be indicated by tinting. The area lying between two selected contours is colored one tint, the area between two other contours another tint, and so on. The areas to be flooded by the construction of dams of different heights, for example, might be shown in different tints.

15-4 Contour Lines

The configuration of the ground and the elevations of points are most commonly represented by means of contour lines, because contours give a maximum amount of information without obscuring other essential details on the map. Some of the principle of contours are represented in Fig. 15-2. Four different contour intervals are shown in views (a), (b), (c), and (d). The steepness of the slopes can be determined from the contour interval and the horizontal spacing of the contours. If all four of these sketches are drawn to the same scale, the ground slopes are the steepest in (d), where the contour interval is 20 ft, and are the flattest in (c), where the interval is 1 ft.

The elevation of any point not falling on a contour line can be determined by interpolating between the two contour lines that bracket the point. Quite often, when the scale of the map is large and the terrain is flat, the successive contours are spaced so far apart horizontally that interpolation between adjacent contours does not have much significance. Therefore, in such an instance, the accuracy and utility of the map is greatly increased by showing the elevations of points at regular intervals in some form of a grid pattern. Elevations between these points are then determined by interpolation. Spot elevations are shown on Fig. 15-3.

A contour cannot have an end within the map. It must either close on itself or commence and end at the edges of the map. A series of closed contours represents either a hill or a depression. From the elevations of the contour lines shown, a hill is represented in Fig. 15-2(a) and a depression in view (b). As indicted, a depression contour is identified by short hachures on the downhill side of the contour. A ravine is indicated by the contours in Fig. 15-2(c). If the elevations were reversed, the same contours would represent a ridge. View (e) is incorrect, as two contours

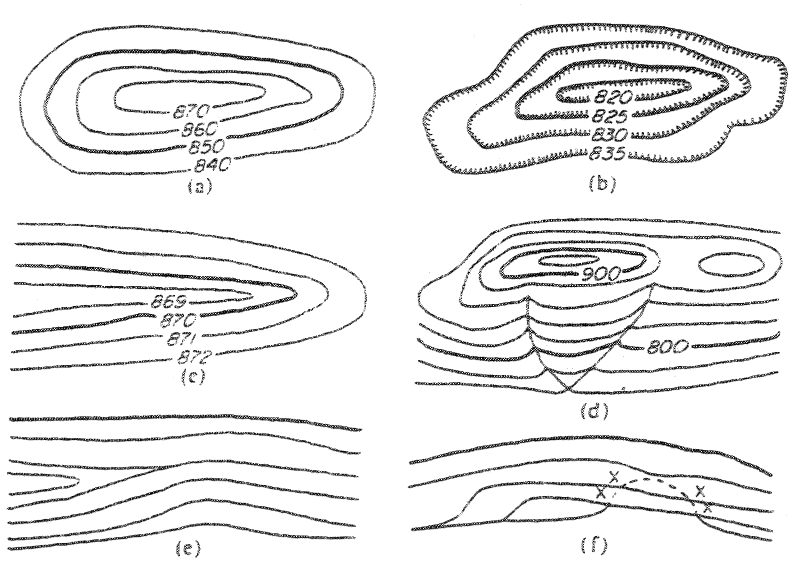

FIGURE 15-2 Contour lines

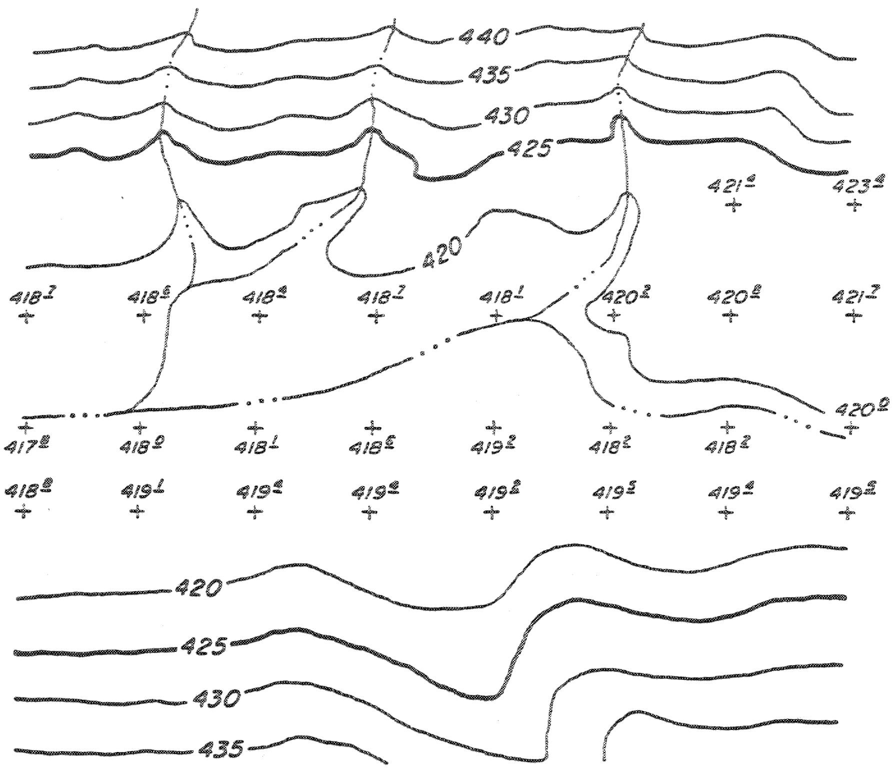

FIGURE 15-3 Spot elevations to supplement contour lines.

are shown meeting and continuing as a single line; this would represent a knife-edged ridge or ravine, something not found in nature. View (f), if not incorrect, is at least unusual. Several contours are shown merging and continuing as a single line. This would be correct only in the case of a vertical slope or a retaining wall. Also, one contour is shown to cross two others. Thus each point marked x has two elevations, a condition found only at a cave or an overhanging cliff.

A series of equally spaced contour lines represents a constant slope along a line normal to the contours. A series of straight, parallel, equally spaced contours represents man-made excavations or embankments. The steepest direction from any point on a topographic map is that which runs normal to successive contour lines near the point.

The drainage of the terrain is the primary agent in shaping the topography. Its influence on the shape of the contour lines can be seen in Figs. 15-1, 15-2(d), and 15-3. Note that as contour lines cross gullies or streams or other drainage features, the contour lines form modified Vs pointing upstream. The form of the Vs is determined by the type of underlying soil or rock. In general, if the underlying material is fine grained like a clay soil, the V will be smooth and rounded, and if the material is coarse and granular, the V will be quite sharp.

As a convenience in scaling elevations from a topographic map, each fifth contour is drawn as a heavier line. This is called an *index contour.* When the interval is 1 ft, contours whose elevations are multiples of 5 ft are shown heavy. When the interval is 10 ft, the heavy contours have elevations that are multiples of 50 ft. Enough contours should be numbered to prevent any uncertainty regarding the elevation of a particular contour. Where the contours are fairly regular and closely spaced, only the heavy contours need be numbered.

15-5 Field Methods

Among the factors that influence the field method to be employed in the compilation of a topographic map are the scale of the map, the contour interval, the type of terrain, the nature of the project, the equipment available, the required accuracy, the type of existing control, and the extent of the area to be mapped. The area to be mapped for highway or railroad location and design takes the form of a strip with a width varying from 100 ft to perhaps more than 1000 ft (30 to 300 m). The control lines are the sides of a traverse that have been established by a preliminary survey and that have been stationed and profiled as outlined in Chapter 3. The method of locating topography most commonly employed for this purpose is the *cross-section* method.

To make an engineering study involving drainage, irrigation, or water impounding or to prepare an accurate map of an area having little relief, each contour line must be carefully located in its correct horizontal position on the map by following it along the ground. This is the *trace contour* method.

When an area of limited extent is moderately rolling and has many constant slopes, points forming a grid are located on the ground and the elevations of the grid points are determined. This is the *grid* method of obtaining topography.

If the area to be mapped is rather extensive, the contour lines are located by determining the elevations of well-chosen break points from which the positions of points on the contours are determined by computation. This is known as the *controlling-point* method.

15-6 Cross-Section Method

The cross-section method of obtaining topography can be performed by transit and tape, by transit stadia, by level and tape, by a combination EDM-theodolite, or by a total station instrument. Horizontal control is established by a theodolite-tape traverse, by an EDM-theodolite traverse, or by a traverse run using a total station instrument. This traverse is run between fixed control points. Stakes are set every 50 or 100 ft or at other pertinent intervals, the spacing depending on the terrain. Vertical control is obtained by profile leveling which may be performed either before the topography is taken or concurrently with the cross sectioning. When using the total station instrument, vertical control is obtained as the traverse progresses.

When the theodolite and tape are used, the instrumentman occupies each station or plus station on the line. He determines the HI by holding the leveling rod alongside the instrument. A right angle is turned off the line, and the rodman, holding one end of the tape, proceeds along this cross line until a break in the slope occurs. If possible, the instrumentman takes a level sight on the rod and records the reading and the distance to the point. If the rod cannot be sighted with the telescope level, a vertical angle is read and the slope distance is recorded. The rise or fall of the line of sight equals the slope distance times the sine of the vertical angle, as discussed in Section 3-3. The horizontal distance to the point is obtained by Eq. (2-1). The rod reading must be taken into account in determining the elevation of the point, as discussed in Section 14-7.

The rodman proceeds to the next break in the topography, and the process is repeated. If the width of strip on each side of the line is greater than the length of the tape, a third man is necessary to hold the tape. As the distance out becomes greater, the slope distance cannot be measured directly in case vertical angles must be read, and the true rise or fall of the line of sight will be somewhere between the measured distance times the sine of the vertical angle and the measured distance times the tangent of the angle.

On a large project in which the width of the project is greater than the length of a tape, the EDM attached to the theodolite or else the total station instrument is employed to advantage. Either of these systems measures the horizontal distance to the point and the difference in elevation between the instrument setup and the point on the cross section. The reflector is held on a staff or rod that contains a level bubble to bring the rod vertical. The height of the reflector above ground together with the HI of the instrument are then used in determining the difference in elevation to the point.

Table 15-1 shows the data recorded for nine regular cross sections spaced at 50-ft intervals, and additional ones at station 22 + 28 and station 23 + 70. The center line elevations are obtained either from prior profile leveling or else by leveling carried along as the cross-section measurements are made. If by the latter method, then profile-level notes would be recorded on the left side of the field book in combination with the cross-section notes on the right side. The cross-section notes consist of the station number, the horizontal distance to the right or left of the center line to the point as the denominator, and the elevation of the point as the numerator. Note that the stationing increases from the bottom upward. A glance at the right side shows that when someone views the notes, he can imagine looking in the forward direction of the line, with the values to the left of him entered as such as those to the right of him entered as such.

If the instrument used in measuring cross sections incorporates an electronic field book or a data collector, the instrumentman then needs only to key in the station number and the direction left or right from the centerline. The distance to and the elevation of the point are automatically stored in the data collector, and are available in the office for plotting. This eliminates the recording of cross-section notes in the field.

The total station instrument can be used in an altogether different manner than that in which each station is occupied, provided that the center line is placed on a coordinate system. This implies that the azimuth of the center line is known and that the coordinates of the stations are also known. The instrument is set up on a station, and the azimuth of the center line together with the coordinates and elevation of the

TABLE 15-1 Recorded Data for Nine Cross Sections

Station	Center Line Elevation	Cross Sections						
			L		C		R	
26 + 00	550.0			550.2/93	550.0/0	547.6/23	539.0/61	532.4/106
25 + 50	534.5			545.0/107	534.5/0	532.1/59	527.7/109	
25 + 00	539.3		551.3/103	549.1/31	539.3/0	537.3/10	521.7/100	
24 + 50	544.2		551.0/94	555.1/52	544.2/0	532.3/52	524.5/106	
24 + 00	551.0		550.0/101	556.3/65	551.0/0	529.1/100		
23 + 70	545.0	548.0/100	557.3/67	553.9/45	545.0/0	539.0/27	528.6/100	
23 + 50	546.6		547.4/102	557.8/61	546.6/0	525.8/94		
23 + 00	550.0		552.0/108	559.8/69	550.0/0	547.6/20	532.0/109	
22 + 50	542.7		554.8/89	549.2/47	542.7/0	538.0/46	535.0/102	
22 + 28	539.5			550.0/92	539.5/0	530.0/62	530.7/96	
22 + 00	539.3	546.6/92	545.5/61	541.4/19	539.3/0	539.7/32	528.6/102	

station are keyed in. The reflector is set to the HI of the instrument. A sight is taken along the center line to orient the circle. The person on the reflector then must remain on or near the perpendicular to the center line when selecting breaks in the topography. When a sight is taken to the reflector, the data collector computes the coordinates and elevation of the point and stores it for future plotting. In this manner several cross sections can be measured from a single instrument setup.

In any of the foregoing methods the located points are plotted in the office. The positions of the contour lines are obtained by interpolating between the elevations of the plotted points as discussed below in Section 15-7.

In compiling topography by the cross-section method, the positions of all planimetric features, such as buildings, fences, streams, and property lines, must be located with respect to the control line and plotted on the topographic map. Methods of locating the positions of points with respect to a traverse line are discussed in Section 6-13.

15-7 Methods of Interpolating

In locating contours on a map by interpolation, the positions of points on the contours can be determined either mathematically, mechanically or with software. In Fig. 15-4 it is required to locate the 5-ft contours on the line connecting points a and b, whose elevations are 873.4 and 896.2, respectively. It is evident that contours at elevations 875, 880, 885, 890, and 895 will cross this line.

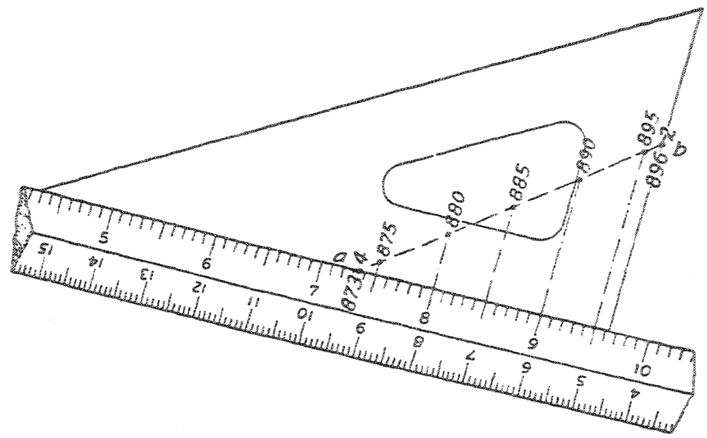

FIGURE 15-4 Interpolating with triangle and scale.

The distances on the map from the point a to the contours can be calculated by proportion. The horizontal distance between a and b on the map is scaled and found to be 2.78 in. The corresponding vertical distance on the ground is 896.2 − 873.4 = 22.8 ft. The vertical distance from a to the 875-ft contour is 875.0 − 873.4 = 1.6 ft. The horizontal distance from a to the 875-ft contour is (2.78/22.8) 1.6 = 0.20 in. The distance between two adjacent 5-ft contours is (2.78/22.8) 5 = 0.61 in. The horizontal distances can also be expressed in terms of the distances on the ground.

Figure 15-4 illustrates a method by which the points on the contours can be located mechanically by using a triangular engineer's scale and a small celluloid triangle. The method is an application of the geometric method of dividing a line into any number of equal parts. The 7.34-in. mark on the scale is pivoted on a, whose elevation is 873.4 ft; the corner of the triangle is placed at the 9.62-in. mark; and both the scale and the triangle are turned until the edge of the triangle passes through b, whose elevation is 896.2 ft. The scale is then held in place while the corner of the triangle is moved successively to the 9.50-, 9.00-, 8.50-, 8.00-, and 7.50-in. points on the scale. Where the edge of the triangle crosses the line ab in the various positions are the corresponding contour points.

This method is very rapid and accurate and entirely eliminates mathematical computations. Any edge of the triangular scale can be used, provided the length on the scale corresponding to the difference in elevation between the two plotted points is shorter than the length of the straight line between the points on the map. When the difference in elevation is considerable and the map distance is short, it may be necessary to let one division on the scale represent several feet in elevation. Thus the smallest division on the scale may correspond to a difference in elevation of 10 ft, instead of 1 ft as in Fig. 15-4. By changing the value of a division, some side of the scale can always be used, regardless of the difference in elevation or the length of the line on the map.

Instead of the triangle and scale, a piece of tracing cloth, on which equally spaced horizontal lines have been ruled, can be used in exactly the same manner. The tracing cloth is turned until lines corresponding to the given elevations pass through the two points on the map. The contour points are then pricked through the tracing cloth to the map beneath.

In most cases the position of the contour crossings can be estimated with sufficient precision, and exact interpolation is unnecessary. When the positions of the contour lines have been plotted, the contour lines are sketched in freehand by joining points at the same elevation.

The cross-section notes shown in Table 15-1 are used to plot the contour lines shown in Fig. 15-5. The center line is first laid out in the proper position on the

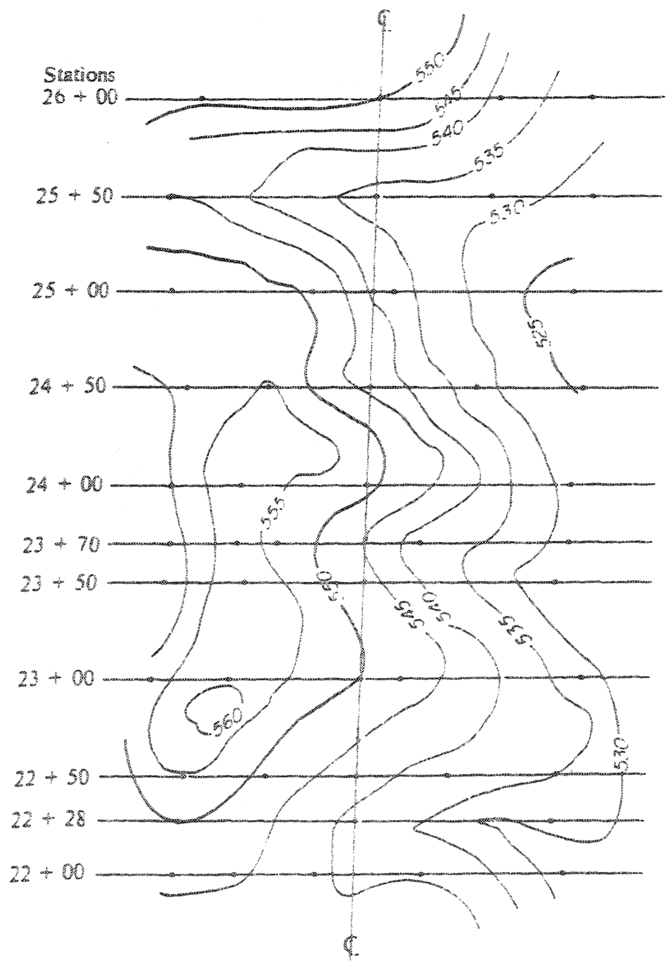

FIGURE 15-5 Contour lines interpolated from cross-section notes.

map sheet, usually by the coordinates of its points of intersection. The center line tangent is then stationed, and cross-section lines are drawn through the station marks. Distances are laid out to the points where elevations were determined by reference to the cross-section notes. These points are represented by small dots in the illustration. Elevations are entered at the points. The positions of the contour-line crossings are then determined; and, finally, the contour lines are sketched in and numbered. Notice than an extra cross section was needed at station 22 + 28 to pick up the stream bed, and at station 23 + 70 to locate the swale in the side of the hill.

If a total station instrument is used to obtain cross sections, the data held in the data collector are downloaded to the control unit of an automatic drafting machine. Then the points are plotted according to their coordinates. Contour lines can then be drawn as discussed previously, or they are located by any of several computer programs used to generate contour lines.

15-8 Trace Contour Method

Under certain critical conditions the surveyor is required to locate contours with a higher degree of accuracy than can be obtained by other field methods. This requires that the positions of a string of points at constant elevation must be determined for each contour line. A network of control points is established in the area to be mapped. The elevations of the control points are determined by leveling or as part of the control survey.

Two methods are discussed here for tracing out contour lines. In the first method a theodolite and leveling rod are employed. In Fig. 15-6 the theodolite is set up at control station S, whose elevation is 232.6 ft. A backsight is taken on control station R, and the horizontal circle is read (or stored). The height of the instrument from the control station up to the horizontal axis of the theodolite is measured (5.2 ft in Fig. 15-6), and added to the elevation of the station to give the elevation of the line of sight of 237.8 ft.

To locate the distance and direction to a point on the 236-ft contour line, the rodman is so positioned as to give a horizontal line of sight reading of 1.8 ft (237.8 − 1.8 = 236). A stadia interval is read to determine the distance to the point. The horizontal circle is then read (or stored). In a similar manner the rodman moves along the contour line with the rod reading of 1.8 ft always maintained. The stadia interval and horizontal circle reading are read and stored for each point. This results in a string of points on the 236-ft contour line.

The 234-ft contour line is located in a similar manner with the rod reading of 3.8 ft (237.8 − 3.8 = 234). This procedure is continued for each subsequent contour line until the instrumentman can no longer read the rod. A new setup at a different elevation is then required.

In the second method a total station instrument with continuous tracking capability is used to advantage in that a larger difference in elevation is no impediment that exists with the leveling rod. In Fig. 15-7 the instrument is set up at the same station S as before. A backsight is taken on station R with the azimuth of SR

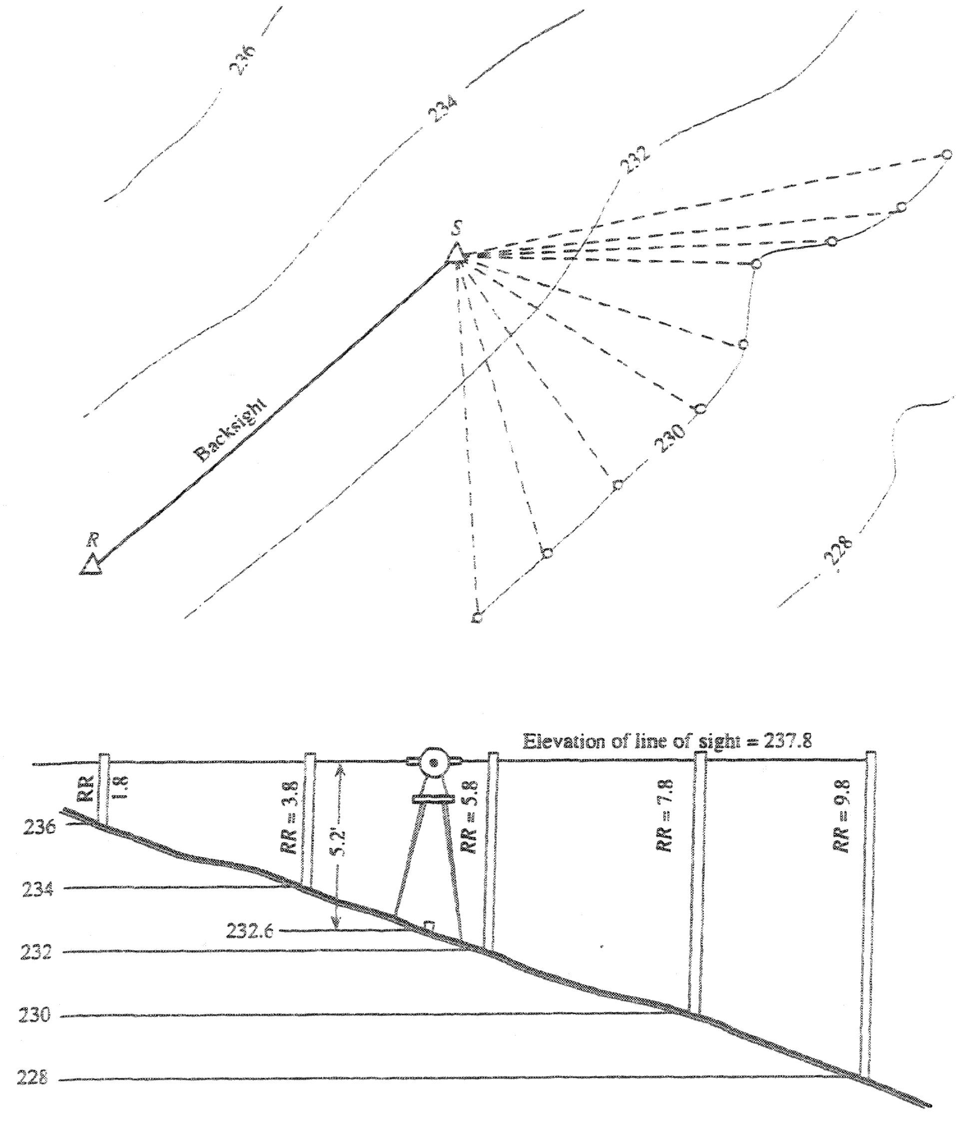

FIGURE 15-6 Trace contouring using theodolite and leveling rod.

entered. The EDM reflector is set to the height of the horizontal axis (5.2 ft in this example). The elevation of station S is entered into the instrument before measurements begin. With the instrument set to the tracking mode, the reflector is moved until the elevation scale reads 236 ft. This puts the foot of the reflector rod on the 236-ft contour line. The horizontal circle (the azimuth), the horizontal distance, and the elevation for this point are all stored. This procedure is repeated for the string of points on each contour line. Since the line of sight can be raised or lowered, there is no such thing as "running out of rod."

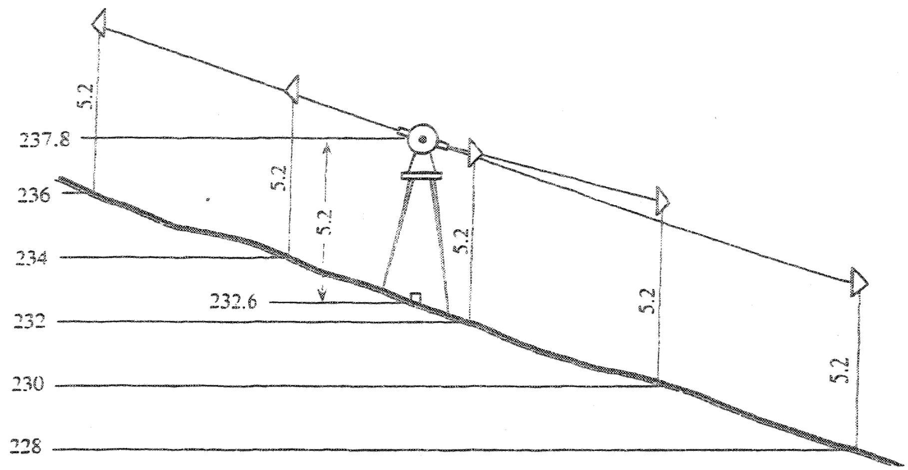

FIGURE 15-7 Trace contouring using a total station instrument

Plotting of the contour lines can be performed using a 360° protractor and scale or else, in the case of the total station instrument, its data can be downloaded to an automatic plotter.

15-9 Grid Method

The grid method of obtaining topography may be used in areas of limited extent where the topography is fairly regular. A level is usually used for determining elevations of the grid points, although a theodolite can be used by bringing the telescope horizontal for each sighting.

If the boundary of the area to be mapped has not been previously surveyed, the first step is to run a traverse around the area and establish the corners. Then, to determine the topography, the area is usually divided, as far as possible, into squares or rectangles of uniform size. The dimensions of these divisions depend on the required accuracy and the regularity of the topography but are usually between 25 and 100 ft. The form chosen for the divisions will depend somewhat on the shape of the area. The size of the divisions should be such that, for the most part, the ground slopes can be considered uniform between the grid points at the corners of the divisions. The grid points either are defined by stakes or are located by ranging out and measuring from stakes already set.

Figure 15-8 represents a tract of land of which a topographic map is to be prepared. It is assumed that a traverse survey locating the boundaries has already been made. The tract is to be divided by means of lines running in two perpendicular directions and spaced 100 ft apart. When a tract is divided in this manner, it is customary to designate by letters the lines that extend in one direction, and by figures the lines at right angles to that direction. The point of intersection of any two lines is then designated by the letter and figure of the respective intersecting lines. The intersections of the dividing lines with the exterior boundary lines on the far

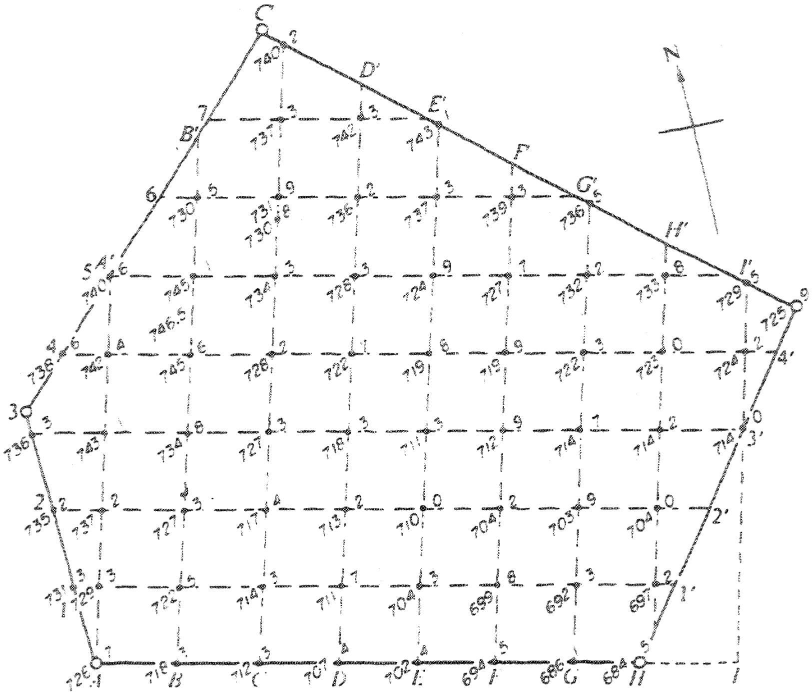

FIGURE 15-8 Topography by grid method.

sides of the tract are designated by the proper letters or numbers affected with an accent or a subscript, as *A'*, *B'*, *2'*, *3'*.

Different methods may be followed in laying out the tract. Any method is satisfactory if it accurately defines the positions of the points of intersection so that they can be readily located when the levels are being taken. For this purpose it is not usually necessary to mark all points of intersection, but enough points should be marked by stakes to permit the remaining points to be located easily and quickly by merely ranging them in from the points that are marked. The rougher and more irregular the surface of the tract, the more stakes must be set. Also, a tract of irregular form usually requires a comparatively greater number of stakes than a tract of rectangular form.

After the required stakes have been set, levels are taken over the tract to determine the elevations at all points of intersection and also at any intermediate points where the slope changes abruptly. Such an intermediate point is generally located in a direct line between two intersections by its distance from the intersection having the lower letter or number. This distance is measured with a tape, approximated by pacing, or merely estimated by the eye, according to the conditions and to the degree of accuracy required, and is recorded as a plus. Thus on line *CC'* there is a low point 80 ft beyond stake *C*5 and its elevation is 730.8; this point would be designated by *C*5 + 80. The high point, whose elevation is 746.5 and which is situated between lines 4 and 5 and between lines *A* and *B*, would be designated in the notes as $A + 80$, $4 + 45$. The levels should be taken in the order that is most advantageous for the nature of the ground. The object is to take rod readings at each of the intersections and at other points with as few settings of the level as possible. To be sure that rod readings are taken at all the intersections, those taken from each setting are checked off on the sketch, or sketches, in which they are all shown.

An EDM mounted on a theodolite or a total station instrument can be used in place of the level. A point that commands a view of a substantial portion of the area to be mapped is located with reference to the control that is used to establish the grid intersections. Its elevation is also determined. After the grid has been staked out, the point is occupied by the EDM instrument, and the reflector is held at each grid intersection. The slope distance to the grid point is measured along with the vertical angle from which the elevation of the grid point can be determined. To locate planimetry, the theodolite is oriented in azimuth by backsighting on a control point, and then the upper motion is used to determine the direction to each planimetric feature. The EDM is used to measure the distance to the point.

After the field work has been completed, the control points, tract boundary, and grid are plotted to the desired scale. The values of the elevations of the grid intersections are then written at the corresponding map positions of the intersections. The positions of the contour lines are located and sketched by interpolation between the grid intersections. Finally, the planimetric features are plotted using a 360° protractor, as discussed in Section 15-8. If a total station instrument is used, the data from the data collector are downloaded into a drafting machine (see Section 8-33) which then plots the points and enters the elevations automatically. Programs are available which automatically generate contour plots from the data.

15-10 Controlling-Point Method

The compilation of a topographic map by determining the positions and elevations of carefully selected controlling points is applicable to nearly every condition encountered in mapping. It is the method used most extensively in mapping a large area to a relatively small scale because of the economy realized. This method can be applied, instead of the cross-section method, to the mapping of a strip of terrain for route-location studies.

The controlling-point method has in the past employed the plane table and alidade. However, the capability of the total station instruments for storing data and downloading into automatic drafting instruments for the generation of contour lines and planimetry has all but displaced the use of the plane table.

A system of control, both horizontally and vertically, is established in the area to be mapped. This presumes that the coordinates and elevations of the control points have now been determined. The total station instrument occupies a control point, the HI is determined, the coordinates of the point have been entered, and a backsight is taken along a line of known azimuth. The person on the reflector now chooses critical points on the terrain that will most closely reflect the true shape of the topography. These points are along ridges, streams, gullies, draws, the top and bottom of constant slopes in the terrain, peaks and depressions—that is, those points that would permit interpolation to determine the positions of contour lines. As the coordinates and elevations of these points are located, the instrumentman must designate the type of point observed to be understood by the program that operates the drafting machine. Ridges, valleys, streams, and gullies are referred to as *break lines* because the terrain changes slope abruptly along these lines.

All planimetric features are located and identified by the unique codes associated with the instrument system being used. Some of the more advanced instruments allow the operator to designate how points are to be joined together just as a draftsman would logically join points.

After all of the control points in the area have been occupied, the data collector is downloaded into the control unit of the plotter. The planimetric features are automatically drawn to the desired scale. The contours are generated by various computer programs, the details of which can be found in the manufacturer's literature. Figure 15-9 illustrates a topographic map generated by an automatic plotter from data downloaded from a data collector. The tick marks are rectangular coordinate grid intersections plotted by the machine.

The controlling-point method depends to a great extent on the reflector person's knowledge of land shapes, slopes, and stream gradients, and his ability to decide where to select points so that he selects neither too many nor too few points. A distinct advantage of the "one-person" instrument described in Section 4-24 is that the instrumentman has total control over the selection of points to be located.

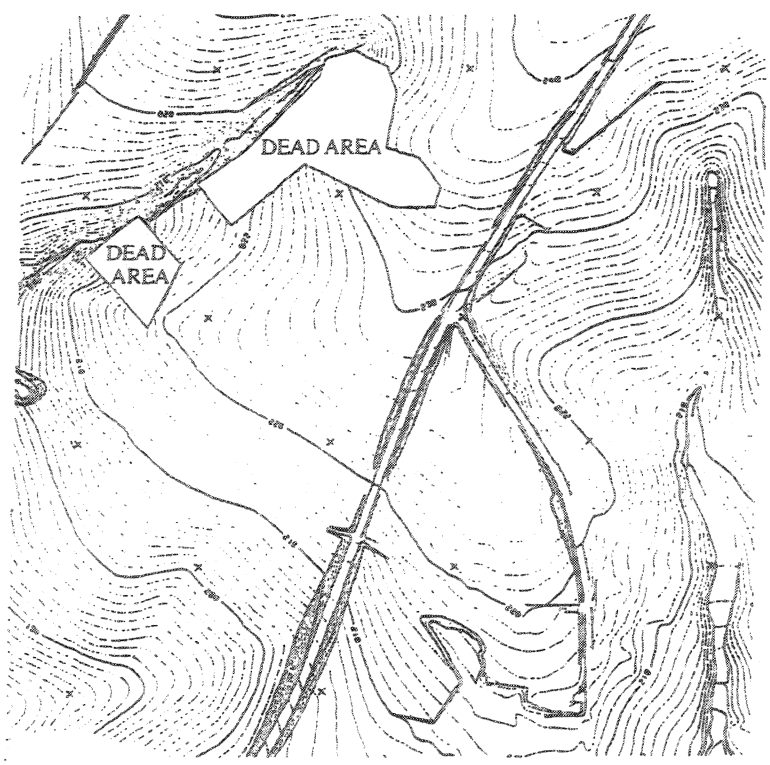

FIGURE 15-9 Topographic map generated from output of data collector.
By permission of Leitz, Inc.

PROBLEMS

15-1. A 100-ft grid is located over an area that measures 800 ft in the north-south direction and 800 ft in the east-west direction. The southwesterly corner is designated *A*-1 and the southeasterly corner is designated *I*-1. The northwesterly corner is designated *A*-9 and the northeasterly corner is designated *I*-9. Elevations are obtained at the grid points using a total station instrument, rounded off to the nearest foot. These are tabulated as follows:

9	375	372	366	360	354	350	340	322	302
8	372	372	370	368	365	362	361	339	312
7	366	365	365	366	364	362	361	347	319
6	363	353	350	360	362	359	350	347	328
5	361	340	335	343	352	350	338	334	328
4	360	344	329	328	337	339	325	321	321
3	360	343	320	317	318	318	313	312	312
2	358	343	323	316	310	308	303	302	303
1	353	340	326	318	310	303	297	292	291
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*

Plot a grid to a scale of 1 in. = 200 ft and draw in 10-ft contour lines.

15-2. Repeat Problem 15-1 with the following values:

9	493	490	488	491	492	493	491	486	483
8	480	474	473	477	484	490	487	480	477
7	479	467	457	460	476	482	479	474	468
6	478	468	459	450	462	470	470	468	461
5	478	470	459	440	440	452	451	460	450
4	480	467	451	438	434	430	427	420	430
3	476	465	459	447	440	433	425	415	409
2	475	472	466	460	454	450	440	422	402
1	472	472	470	468	465	462	461	439	412
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*

15-3. Repeat Problem 15-1 with the following values:

9	586	583	579	570	560	550	528	520	509
8	580	577	577	574	570	563	558	550	542
7	574	568	568	570	570	565	562	557	551
6	568	561	556	561	564	559	553	550	552
5	560	550	537	546	561	550	539	540	547
4	520	530	522	534	544	537	523	520	536
3	515	509	514	525	532	531	520	510	518
2	522	502	500	507	520	525	520	508	507
1	539	512	494	483	490	510	517	506	498
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*

15-4. Repeat Problem 15-1 with the following values:

9	722	702	700	707	720	725	720	708	707
8	739	712	694	683	690	710	717	706	698
7	747	719	700	685	676	690	707	700	689
6	747	728	703	688	670	673	690	687	679
5	734	728	707	690	673	659	664	670	669
4	721	721	710	692	674	660	650	653	658
3	712	712	707	693	671	657	647	639	642
2	702	703	700	683	664	651	641	634	636
1	692	691	682	668	655	641	633	626	630
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*

15-5. The cross-section notes on page 546 were taken for the purpose of plotting a strip of topography for road location. The positions of stream crossings are marked (*). Plot the tangent between stations 18 and 27 to a scale of 1 in. = 100 ft, and sketch in each 2-ft contour line.

15-6. The cross-section notes on page 547 were taken for the purpose of plotting contour lines of a strip of topography. Plot the tangent from station 0 to station 3, using a scale of 30 ft to the inch. Plot the notes on a sheet of $8\frac{1}{2} \times 11$-in. paper, and sketch the 5-ft contour lines. Show the location of the stream (*).

15-7. The elevations for the gridded area shown in Fig. 15-10 are as follows:

F	690	709	726	732	735	740	743	747
E	696	715	729	736	740	737	739	740
D	702	723	733	742	737	737	745	737
C	707	726	732	732	729	740	740	730
B	709	725	723	722	730	738	731	727
A	711	721	715	715	732	730	719	722
	1	2	3	4	5	6	7	8

$P = 741 \qquad Q = 751$

Construct a 1-in. grid on a sheet of $8\frac{1}{2} \times 11$-in. paper and sketch in the 5-ft contour lines. Label and broaden every fifth contour line.

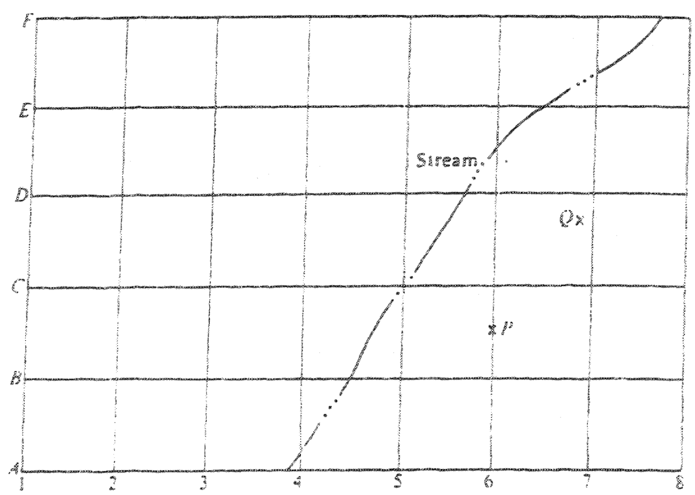

FIGURE 15-10 Grid for solution to Problem 15-7.

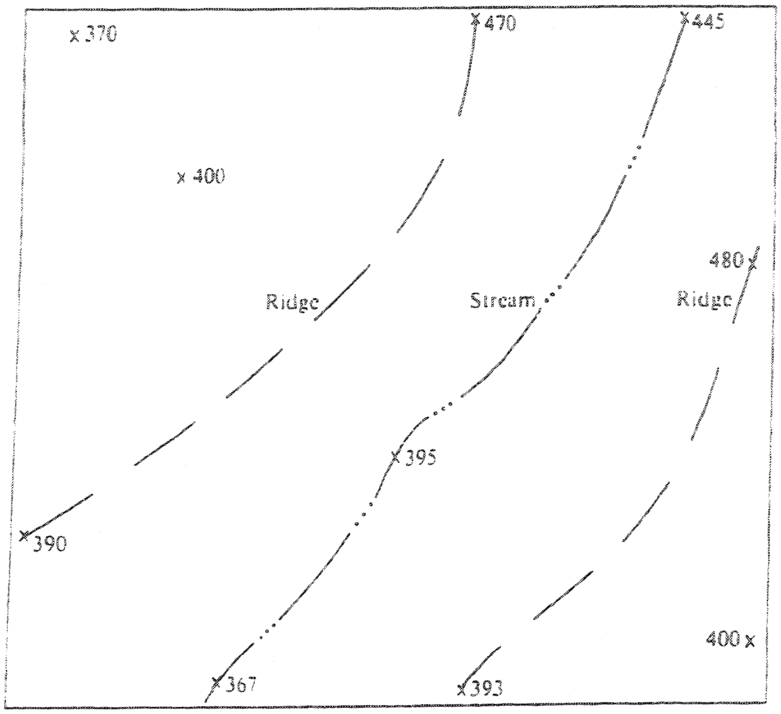

FIGURE 15-11 *Diagram to accompany Problem 15-8*

15-8. The controlling points shown in Fig. 15-11 were obtained by a total station instrument. Trace the figure and sketch the 10-ft contour lines. Number and broaden every fifth contour line, including the 400-ft line.

Cross-Section Notes

Station		Left		Center Line		Right	
27 + 00	$\frac{473.1}{200}$	$\frac{461.4}{107}$	$\frac{450.08}{53}$	$\frac{450.4}{0}$	$\frac{439.4}{200}$		
26 + 50	$\frac{458.0}{200}$	$\frac{449.8}{97}$	$\frac{429.3}{17}$	$\frac{427.7}{0}$	$\frac{425.9}{116}$	$\frac{421.3^*}{200}$	
26 + 00		$\frac{445.2}{200}$	$\frac{428.2}{65}$	$\frac{425.7}{0}$	$\frac{425.3^*}{8}$	$\frac{423.6}{94}$	$\frac{421.9}{200}$
25 + 93				$\frac{425.5^*}{0}$			
25 + 50		$\frac{430.8}{176}$	$\frac{429.0}{140}$	$\frac{428.6}{0}$	$\frac{424.3}{154}$	$\frac{422.0}{200}$	
25 - 00		$\frac{436.1}{200}$	$\frac{430.8}{91}$	$\frac{431.0}{0}$	$\frac{430.3}{65}$	$\frac{423.2}{200}$	
24 + 90				$\frac{431.6}{0}$	$\frac{422.5^*}{200}$		
24 + 50			$\frac{439.3}{200}$	$\frac{432.4}{0}$	$\frac{425.0^*}{136}$	$\frac{424.8}{200}$	

Cross-Section Notes—*continued*

Station	Left			Center Line	Right		
24 + 00		443.2/200	435.8/93	432.1/0	427.7*/94	428.1/115	425.8/200
23 + 76		444.9/200	433.8/73	431.4*/0	426.9/200		
23 + 15		438.4*/182	438.0/150	437.5/0	437.5/80	431.2/200	
23 + 00		440.0/200	438.1/87	439.2/0	438.8/113	432.1/200	
22 + 00		448.9/200	448.2/164	450.0/0	439.8/139	433.7/200	
21 + 00	459.9/200	460.0/153	457.2/88	446.8/0	433.4/102	424.6/200	
20 + 00			467.0/200	438.6/0	420.7/143	416.1/200	
19 + 00			479.2/200	443.5/0	422.4/111	414.0/200	
18 + 00	499.1/200	475.4/118	461.2/63	447.0/0	433.2/80	419.2/200	

Cross Section Notes

Station	Left							Center Line	Right					
3 + 00	405/50	400/43	395/32	390/17	385/3			384/0	380/18	376*/35				
2 + 50	400/46	395/35	390/25	385/9				382/0	380*/8	385/39	390/50			
2 + 00	400/47	395/35	390/27	385/20	384*/10			385/0	390/5	395/11	400/20	400/33	395/50	
1 + 89								390/0						
1 + 78								395/0						
1 + 69								400/0						
1 + 62								405/0						
1 + 54								410/0						
1 + 50	400/50	395/42	390*/32	395/19	400/13	405/9	410/4	412/0	415/6	415/12	410/18	405/27	400/33	395/50
1 + 42								415/0						
1 + 25								417/0						
1 + 00	400/50	405/44	410/36	415/24				416/0	415/4	410/10	405/21	400/30	395/48	

Cross Section Notes—*continued*

Station	Left	Center Line	Right
0 + 95		$\frac{415}{0}$	
0 + 84		$\frac{410}{0}$	
0 + 68		$\frac{405}{0}$	
0 + 50	$\frac{415}{50}$ $\frac{410}{33}$ $\frac{405}{14}$	$\frac{403}{0}$	$\frac{400}{11}$ $\frac{395}{27}$ $\frac{390}{50}$
0 + 32		$\frac{400}{0}$	
0 + 00	$\frac{415}{52}$ $\frac{410}{44}$ $\frac{405}{33}$ $\frac{400}{14}$	$\frac{397}{0}$	$\frac{395}{7}$ $\frac{390}{28}$ $\frac{385}{48}$

15-9. Reduce the following transit-stadia notes, and plot the positions of the points to a scale of 1 in. = 20 ft. Sketch 1-ft contour lines. Assume $K = 100$ and $C = 0$. Line AB is due north.

Transit at B, Elevation = 459.5 ft BS 0° 00′ on A						
Point	Interval	Horizontal Distance	Azimuth	Vertical Angle	DE	Elevation
1	0.56		0° 00′	−1° 20′		
2	0.99		1° 00′	−0° 28′		
3	0.25		10° 15′	+4° 36′		
4	0.85		12° 15′	+1° 58′		
5	1.06		16° 45′	−0° 03′		
6	0.58		17° 45′	−2° 22′		
7	0.76		30° 00′	−1° 49′		
8	1.08		40° 45′	−2° 14′		
9	1.43		45° 00′	−2° 48′		
10	0.83		50° 00′	−3° 15′		
11	0.59		61° 45′	−2° 32′		
12	1.12		61° 45′	−4° 41′		
13	0.86		68° 45′	−4° 05′		
14	1.00		68° 45′	−4° 40′		
15	1.04		75° 15′	−6° 07′		
16	0.91		77° 00′	−6° 21′		
17	0.45		80° 30′	−3° 04′		
18	0.68		83° 30′	−3° 38′		
19	0.80		86° 30′	−6° 12′		
20	1.00		88° 45′	−9° 05′		

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17 Earthwork

17-1 Introduction

Earthwork operations involve the determination of the volumes of materials that must be excavated or embanked on an engineering project to bring the ground surface to a predetermined grade, and the setting of stakes to aid in carrying out the construction work according to the plans. Although the term *earthwork* is used, the principles involved in determining volumes apply equally well to volumes of concrete structures, to volumes of stock piles of crushed stone, gravel, sand, coal, and ore, and to volumes of reservoirs. The field work includes the measurements of the dimensions of the various geometrical solids that make up the volumes, the setting of grade stakes, and the keeping of the field notes. The office work involves the computations of the measured volumes and the determination of the most economical manner of performing the work.

The units used for measurement in earthwork computations are the foot, the square foot, and the cubic yard. In the metric system the corresponding units are the meter, the square meter, and the cubic meter. This chapter employs the foot-yard system. However, metric units can be substituted with equal validity.

Because of the inherent accuracy of modern topographic maps of large scale produced by photogrammetric methods, much of the field work to be discussed in this chapter is eliminated, except for earthwork of limited extent. The measurements for the determination of volumes can be made directly from stereoscopic models or on topographic maps prepared by photogrammetric methods (see Sections 16-9, 17-13, and 17-14).

17-2 Cross Sections

A cross section is a vertical section taken normal to the direction of the proposed center line of an engineering project, such as a highway, railroad, trench, earth dam, or canal. A simple cross section for a railroad embankment is shown in Fig. 17-1. The cross section for a highway or an earth dam would have similar characteristics. It is bounded by a base *b*, side slopes, and the natural terrain. The inclination of a side slope is defined by the horizontal distance *s* on the slope corresponding to a unit vertical distance. The slope may be a rise (in excavation) or a fall (in embankment). A side slope of $3\frac{1}{2}$: 1, for example, means that for each $3\frac{1}{2}$ ft of horizontal distance the side slope rises or falls 1 ft. This can be designated as $3\frac{1}{2}$: 1 or 1 on $3\frac{1}{2}$.

17-3 Preliminary Cross Sections

In making a preliminary estimate and in determining the location of a facility, such as a highway or railroad, a preliminary line is located in the field as close to the final location of the facility as can be determined from a study of the terrain supplemented by maps or aerial photographs of the area. The preliminary line is stationed, and profile levels are taken. The configuration of the ground normal to the line is obtained by determining the elevations of points along sections at right angles to the line. This is identical to the process of obtaining elevations for topographic mapping described in Section 15-6.

The values of the elevations and the corresponding distances out to the right or left of the preliminary line can be plotted on specially printed cross-section paper, at a relatively large scale of from 1 in. = 5 ft to 1 in. = 20 ft (however, see Sections 17-13 and 17-14). When the location and grade of a trial line representing a tentative location of the center line of the facility have been established, the offset distance from the preliminary line to the trial line is plotted, and the grade elevation of this trial line is plotted in relation to the terrain cross section. In Fig. 17-2 the elevations of, and distances to, the points plotted on the ground line were determined with reference to the preliminary center line. These are shown as

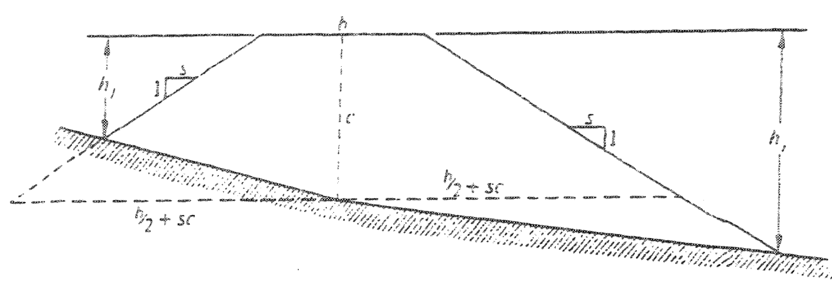

FIGURE 17-1 Cross section in fill for railroad embankment.

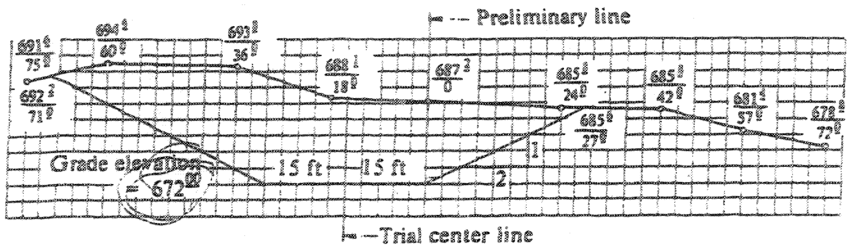

FIGURE 17-2 Preliminary cross section.

fractions, with elevations as the numerators and distances from the preliminary line as the denominators. The offset distance of 15 ft from the preliminary line to the trial line is plotted, and the base of the roadbed is plotted at grade elevation 672.00 ft. At the edges of the roadbed, the side slopes of 2 : 1 are laid off and drawn to intersect the terrain line at the scaled distances and elevations shown as fractions lying under the terrain line. These points of intersection are called *catch points*.

The cross-sectional area bounded by the base, the side slopes, and the ground line of each trial cross section along the trial line is determined from the plotted cross section by using a planimeter, by computation based on the formulas for areas given in section 8-21, or by surveying software. This procedure is discussed in Section 17-8. The volumes of excavation and embankment for this trial line are computed from the successive areas and the distances between the areas by the methods described in Sections 17-9 to 17-11. The volumes for various trial lines are compared. The necessary changes in line and grade are then made to locate the final line and establish the final grade. This location will require a minimum of earthwork costs and, in the case of a highway project, for example, it will at the same time meet the criteria designed for curvature, maximum grade, and safe sight distances.

17-4 Final Cross Sections

The line representing the adopted center line of a facility is staked out in the field and stationed. This line is located by computing and running tie lines from the preliminary line as discussed in Section 8-24. Deflection angles are measured between successive tangents, and horizontal curves are computed and staked out. Reference stakes are sometimes set opposite each station on both sides of the center line at distances of 25, 50, or 100 ft from the center line. These stakes are used to relocate the center line after grading operations are begun. Stakes at a distance on either side equal to half the base width are sometimes driven to facilitate taking final cross sections and setting construction or slope stakes. The center line and the reference lines are then profiled.

When the final line has been located and profiled, a cross section is taken at each station to determine the area of the cross section and at the same time to locate the limits of excavation of embankment. These limits are defined on the ground by stakes. The process of setting these stakes is called *slope staking*.

In Fig. 17-3 the level is set up and a backsight is taken on the leveling rod held at a station on the center line whose elevation has been determined from the profile levels. The HI is established as 880.2 ft. The base width given on the plans is 24 ft, the side slopes are $1\frac{1}{2} : 1$, and the grade elevation (the design elevation of the road surface) at the station is 878.4 ft. If the leveling rod were held so that the foot of the rod were at grade, the reading of the rod would be 1.8 ft. This is equal to the HI minus the grade elevation and is called the *grade rod*. The grade rod can be plus or minus. The vertical distance from a point on the ground line to the grade line at any section is called the *fill* at the point if the section is in embankment, or the *cut* if the section is in excavation. Figure 17-3 shows that, at the left edge of the cross section, the fill is 8.6 ft, at the center line the fill is 6.2 ft, and at the right edge the fill is 4.2 ft.

The grade rod may be determined from the center-line cut or fill and the rod reading at the center line by the following relationship:

$$\text{grade rod} = \text{ground rod} + \text{center cut}$$

or (17-1)

$$\text{grade rod} = \text{ground rod} - \text{center fill}$$

in which the ground rod is the rod reading at the center line. The ground rod in Fig. 17-3 is 8.0 ft. So the grade rod is 8.0 − 6.2 = 1.8 ft, as determined before.

With the grade rod established for the cross section, the amount of cut or fill at any point in the section can be determined by reading the rod held at the point and applying the following relationship:

$$\text{cut or fill} = \text{grade rod} - \text{ground rod} \tag{17-2}$$

If the result is plus, the point is above grade indicating cut (+); if the result is minus, the point is below grade, indicating fill (−).

The location, on the ground, of the slope stake is determined by trial. When the ground surface is horizontal, the position of the slope stake is at a distance from the center line equal to one-half base width plus the product of the side-slope ratio and the center cut or fill. This is shown in the dashed-line portion of Fig. 17-1.

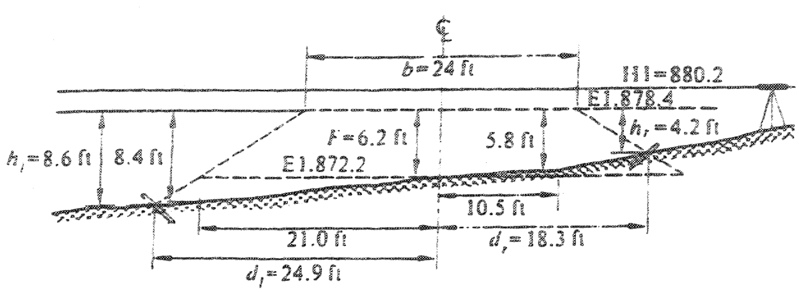

FIGURE 17-3 Slope staking. Side slopes are 1.5 : 1.

If the ground in Fig. 17-3 were horizontal, each slope stake would be located at a distance from the center line equal to

$$12 + 1.5 \times 6.2 = 21.3 \text{ ft}$$

However, since the ground slopes transversely to the center line, it can be seen that the right slope stake will be less than 21.3 ft from the center line while the left slope stake will be greater than 21.3 ft from the center line. By trial, a point is found where 12 ft plus 1.5 times the fill computed by Eq. (17-2) equals the actual distance from the center line. With some experience the point can be found by one or two trials. If it is assumed that the ground appears to rise about 1.5 ft between the center line and the right-hand edge, the fill at the right-hand edge will be 6.2 − 1.5 = 4.7 ft and the distance out should be

$$12.0 + 1.5 \times 4.7 = 19.0 \text{ ft}$$

On the basis of this estimate, the rod is held 19 ft from the center line and a rod reading is taken. If the actual reading at this point is found to be 5.8 ft, then by Eq. (17-2) the depth below the grade is

$$1.8 - 5.8 = -4.0 \text{ ft}$$

To position the point at the intersection of the side slope with the ground surface, the distance from the center should be

$$12.0 + 1.5 \times 4.0 = 18.0 \text{ ft}$$

Since the actual distance was 19.0 ft, the trial point is incorrect. The rod is therefore moved nearer the center. Had the ground surface at the right edge been horizontal, the slope stake could have been set at the 18.0-ft point. Since the ground is sloping downward toward the center line, the fill will be somewhat more than 4.0 ft, and the distance out will be greater than 18.0 ft. Consequently the next trial is taken at 18.2 ft, where a rod reading of 6.0 ft is obtained. The depth below the grade is

$$1.8 - 6.0 = -4.2 \text{ ft}$$

and the distance from the center line should be 18.3 ft. This is within 0.1 ft of where the rod is being held, so the stake is set at 18.3 ft from the center and the fill is recorded as 4.2 ft.

In the field notes these two dimensions are recorded in fractional form, the numerator representing the fill and the denominator representing the distance out from the center. To distinguish between cut and fill, either the letters C and F or the signs + and − are used to designate them. As the point is located with respect to the finished grade, a point below grade indicates a fill and is designated by a − sign. The amount of cut or fill is marked on the side of the stake toward the center stake, and the distance out is marked on the opposite side. The stake is usually driven slantingly to distinguish it from a center-line stake and to prevent it from being disturbed during the grading operations.

In Fig. 17-3 there is a decided break in the ground surface between the center line and the right slope stake. This break is located by taking a rod reading there and measuring the distance from the center line. In a similar manner the break on the left side of the center and the left slope stake are located. The field notes for this particular station could be recorded as shown in the following table:

Station	Grade Elevation	Ground Elevation	Cross Section L		C	R	
15 + 00	878.4	872.2	$\frac{-8.6}{24.9}$	$\frac{-8.4}{21.0}$	$\frac{-6.2}{0}$	$\frac{-5.8}{10.5}$	$\frac{-4.2}{18.3}$

17-5 Cross Sectioning by Slope Measurement

The position of a slope stake can be determined by measuring the distance out from the center line along the slope and determining the difference in elevation between the center-line stake and the tentative position of the slope stake. Figure 17-4 shows a cross section in cut with a roadbed width b and side slope $s : 1$. The cut at the center line is designated c. A plus slope measurement

$$M'P' = t$$

is made from a point M' vertically above the center-line stake at M to a point P' vertically above the tentative position of the slope stake at P. The slope of $M'P'$ is the vertical angle α. In the figure

$$MM' = PP'$$

Then

$$V = t \sin \alpha \text{ and } H = t \cos \alpha$$

The vertical distance V is the difference in elevation between M and P.

If P is the correct position of the slope stake, then the total rise of the side slope is $c + V$ (or $c - V$ on the downhill side). The side slope is in the ratio

$$\frac{1}{s} = \frac{c+V}{D} = \frac{c+V}{H-b/2}$$

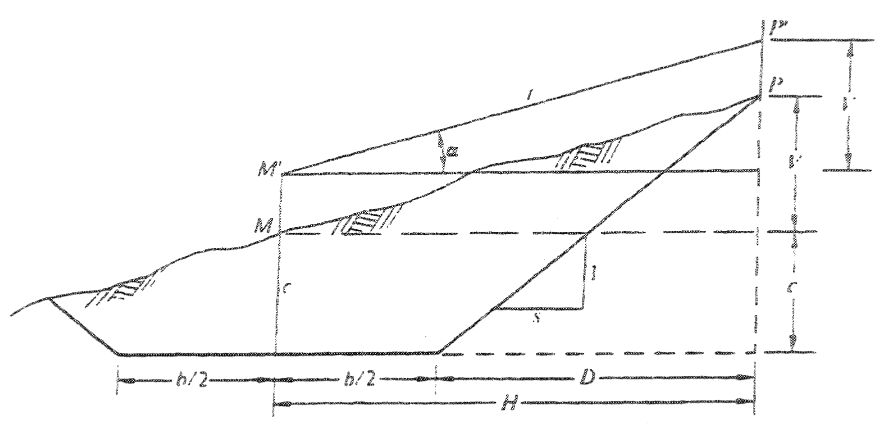

FIGURE 17-4 Slope staking by slope measurements.

Thus considering both an uphill and a downhill sight,

$$H = \frac{b}{2} + s(c \pm V) \tag{17-3}$$

If the computed value of $H = t \cos \alpha$ is the same as the value computed by Eq. (17-3), the slope stake is in the correct position. If

$$H = t \cos \alpha$$

is larger than that obtained by Eq. (17-3), the slope stake is too far out and must be moved in. Thus the final position of the slope stake is determined by trial.

If the cross section is a fill section, then

$$H = \frac{b}{2} + s(f \mp V) \tag{17-4}$$

in which f is the center fill, considered a positive quantity in this equation. If the sight is to the uphill side, the negative sign is used for V, and if downhill the plus sign is used.

Various methods of making the measurements can be used. If a theodolite is set up at M and MM' is measured, the vertical angle α is measured to a point P' on the leveling rod or staff such that

$$PP' = MM'$$

The slope distance is then measured with a steel or metallic tape.

The same procedure as above can be performed using the principle of transit stadia. The values of H and V are those discussed in Section 14-5. The leveling rod held at the slope-stake position should be carefully plumbed using a rod level. This method eliminates the use of the tape.

Several of the EDMs described in Chapter 2 are mounted directly onto transits or theodolites. Others are equipped with vertical circles. These instruments can be used for slope staking in a manner similar to the use of the transit and tape. The reflector is set on a staff at the same height as the horizontal axis of the transit or theodolite or of the instrument itself. The vertical angle α and the slope distance t are measured with the instrument combination. After a trial position of the slope stake has been made, the reflector is moved inward or outward along the cross section according to the difference between the two computed values of H, and a second determination is made. Thus the slope-stake position is fixed by trial and error.

Some EDMs have provisions for computing the values of H and V internally, as discussed in Chapter 2. This eliminates having to do some of the computations in the field as the slope staking progresses. Any of the total station instruments discussed in Chapter 4 can be employed for slope staking since they compute the values of H and V.

17-6 Location of Slope Stakes by Inversing

In modern transportation engineering, all of the surveys are placed on a coordinate system. The preliminary line from which preliminary cross sections are measured (however, see Sections 17-13 and 17-14) is surveyed on the coordinate system. The final alignment is laid out with respect to the control points as discussed in Section 8-24. The preliminary cross sections as shown in Fig; 17-2 are used to compute the positions of the catch points. The base width, side slopes, and grade elevation for each cross section are entered into a computer program. The computer then intersects the side slope lines with the appropriate lines on the preliminary cross section, using Eqs. (8-15) and (8-16). Output from the computer gives the distance to the left (d_L) and to the right (d_R) where slope stakes are to be located, together with the cuts and fills at the catch points and the areas of the cross section. This is described in detail in Section 17-13.

If each station on the final located line is occupied with a theodolite, theodolite-EDM combination, or a total station instrument, a right angle is laid off from the line, and the calculated distance left or right is measured off and the slope stake is set. This, however, requires an instrument setup at each station. If a control point with a commanding view of a substantial portion of the project is convenient, the instrument can be set up at this control point from which angles and distances are measured off to locate several slope stakes.

In Fig. 17-5 the location center line is on a coordinate system. Thus the coordinates of each center-line station are known, as are the coordinates of a control

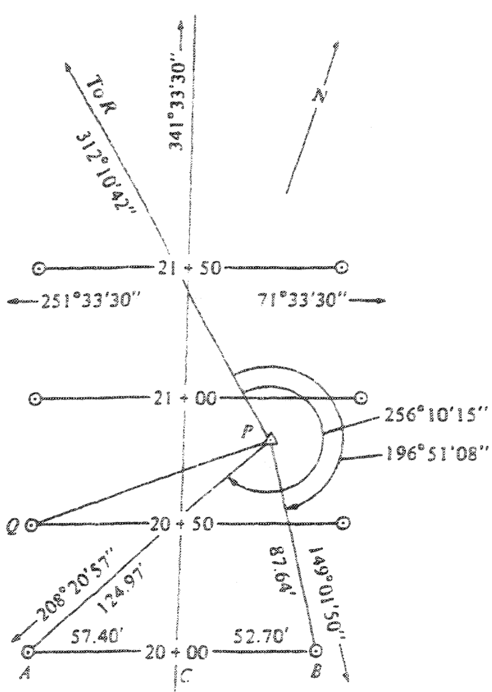

FIGURE 17-5 Location of slope stakes by inversing.

point P. The azimuth of the center line is shown as 341° 33′ 30″. The azimuth of each of the crosslines going to the left of the center line is 90° less, or 251° 33′ 30″. The azimuth of these lines going to the right is 90° more, or 71° 33′ 30″. The latitudes and departures can be computed for each crossline going to the left and to the right since the distances (the d_L's and d_R's) and the azimuths are known. The coordinates of the catch points where the slope stakes are to be set can then be computed based on the coordinates of the center-line stations.

The length and azimuth of any line from the control point P to a catch point such as point Q to the left of station 20 + 50 are computed by Eqs. (8-12) and (8-13). The angles to be laid off at P from a backsight on another control station to each catch point can be computed from the known azimuths. Some of the total station instruments can be programmed to compute the angle and distance to be laid off in order to set the slope stake. The program generally follows the computations shown in Example 17-1.

EXAMPLE 17-1 In Fig. 17-5, the coordinates of the center line at station 20 + 00 (point C) are X = 1,508,697.85 and Y = 457,715.20. The coordinates of control point P are X = 1,508,702.74 and Y = 457,807.02. The azimuth from P to control point R is 312° 10′ 42″ as shown. The distance CA to the slope stake to the left of station 20 + 00 is 57.40 ft; the distance CB to the slope stake to the right of station 20 + 00 is 52.70 ft. Compute the clockwise angle at P from R to A and from R to B, and compute the distance PA and PB.

Solution: The latitudes and departures of CA and CB and the coordinates of A and B are computed as follows:

Point	Length	Azimuth	Latitude	Departure	Y	X
C					457,715.20	1,508,697.85
	57.40	251° 33′ 30″	−18.16	−54.45		
A					457,697.04	1,508,643.40
C					457,715.20	1,508,697.85
	52.70	71° 33′ 30″	+16.67	+49.99		
B					457,731.87	1,508,747.84

Inversing line PA:

X_A = 1,508,643.40 Y_A = 457,697.04
X_P = 1,508,702.74 Y_P = 457,807.02
−59.34 −109.98

azimuth PA = 208° 20′ 57″
length PA = 124.97 ft

Clockwise angle from R to A:

azimuth PA = 208° 20′ 57″
+ 360°
568° 20′ 57″
azimuth PR = 312° 10′ 42″
angle = 256° 10′ 15″

Inversing line PB:

X_B = 1,508,747.84 Y_B = 457,731.87
X_P = 1,508,702.74 Y_P = 457,807.02
+45.10 −75.15

azimuth PB = 149° 01′ 50″
length PB = 87.64 ft

Clockwise angle from R to B:

azimuth PB = 149° 01′ 50″
+ 360°
509° 01′ 50″
azimuth PR = 312° 10′ 42″
angle = 196° 51′ 08″ ◆

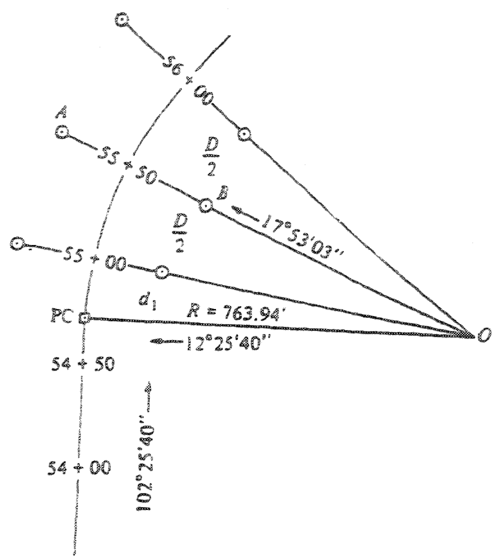

FIGURE 17-6 Location of slope stakes on a curve by inversing.

If the center line is a curve, the cross sections lie on radii from the center of the curve. In Fig. 17-6, if the center line is on a coordinate system, the coordinates of the PC are known. The coordinates of the center of the curve at O can be computed since the radius is known and the azimuth of the radius from the PC to O is (in this example) the azimuth of the center line plus (or minus) 90°. The azimuths of the radii to the curve points can then be determined by adding (or subtracting) the central angles d_1, $d_1 + D/2$, $d_1 + D/2 + D/2$, and so on to (or from) the azimuth of O – PC. The distance from O to any left slope stake is then R plus (or minus) the distance from the center line to the left slope stake; the distance from O to any right slope stake is then R minus (or plus) the distance from the center line to the right slope stake. (The terms in parentheses apply to a curve that deflects to the left.)

EXAMPLE 17-2 In Fig. 17-6 the azimuth of the center line approaching the curve is 102° 25′ 40″. The coordinates of the PC are X = 1,418,065.24 and Y = 434,312.12. The degree of curve is 7° 30′ (arc definition). The radius is 763.94 ft. The PC is at station 54 + 77.25. then d_1 is 1° 42′ 23″ and $D/2$ is 3° 45′. The distance to the left slope stake A at station 55 + 50 is 73.60 ft from the center line and to the right slope stake at B is 83.54 ft. Compute the coordinates of A and B.

Solution: The azimuth from the PC to O is 102° 25′ 40″ + 90° = 192° 25′ 40″. The azimuth from O to the PC is then 12° 25′ 40″. The azimuth from O to station 55 + 50 is 12° 25′ 40″ + 1° 42′ 23″ + 3° 45′ = 17° 53′ 03″. Computation of latitudes, departures, and coordinates are as follows:

Point	Length	Azimuth	Latitude	Departure	Y	X
PC					434,312.12	1,418,065.24
	763.94	192° 25′ 40″	−746.04	−164.41		
O					433,566.08	1,417,900.83
O					433,566.08	1,417,900.83
	837.54	17° 53′ 03″	+797.07	+257.20		
A					434,363.15	1,418,158.03
O					433,566.08	1,417,900.83
	680.40	17° 53′ 03″	+647.52	+208.95		
B					434,213.60	1,418,109.78

The calculation of the coordinates of slope stakes given above for simple circular curves can also be applied to compound and reversed curves and to spirals, taking into account the geometry of these curves.

All of the above computations are carried out for each slope stake in one program in a data processor, and the output is given to the field engineer. The control point is occupied with a combination theodolite/EDM or a total station instrument, all of the catch points that can be seen from the control point are located, and the slope stakes are set. ◆

17-7 Distance Between Cross Sections

The horizontal distance between cross sections is dependent on the precision required, which in turn is dependent on the price per cubic yard paid for excavation. As the unit cost for highway or railroad grading is usually small, a station distance of 100 ft is generally sufficiently precise. For rock excavation and for work done under water, the cost per cubic yard mounts very rapidly, and the distance between sections is often reduced to 10 ft.

In addition to the cross sections taken at regular intervals, other sections are taken at the point of curvature (PC) and the point of tangency (PT) of each curve, at all breaks in the ground surface, and at all grade points. A *grade point* is a point where the ground elevation coincides with the grade elevation. In passing from cut to fill or from fill to cut, as many as five sections may be needed in computing the volume when the change occurs on a side hill. In Fig. 17-7 these sections are located at *A*, *B*, *C*, *D*, and *E*. Unless extreme precision is required, the sections at *A* and *D* are usually omitted. Although stakes are set only at the grade points *B*, *C*, and *E*, full cross sections are taken at the stations and the measurements are recorded in the field notes.

It will be noted in Fig. 17-7 that a wider base of cross section is used when excavation is encountered. The additional width is needed to provide ditches for draining the cut.

17-8 Calculation of Areas

The purpose of cross sectioning is to obtain the measurements necessary to compute the area of the plane of the cross section bounded by the ground, the side slopes, and the roadbed. The cross-section areas are then used to compute

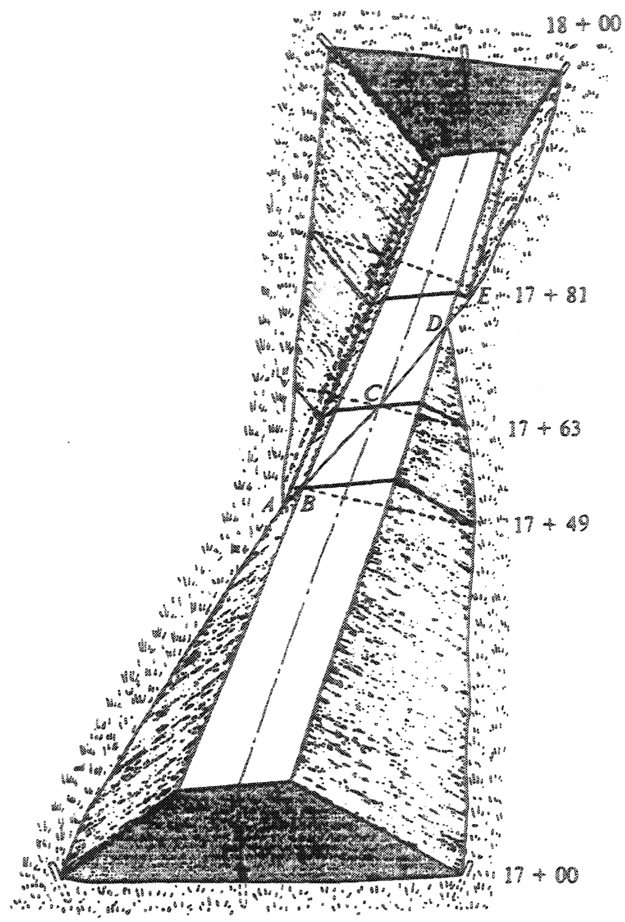

FIGURE 17-7 *Transition from fill to cut.*

earthwork volumes, either by the average end-area method discussed in Section 17-9 or by the prismoidal formula given in Section 17-10.

Four classes of cross-section configurations are shown in Fig. 17-8. The first is the level section shown in view (a). The area of the cross section is given by

$$A = (b + fs)f \tag{17-5}$$

in which b is the width of the roadbed, f is the center fill, and s is the side-slope ratio. If the cross section is in excavation, then c is the center cut and is substituted for f in Eq. (17-5).

Figure 17-8(b) shows a three-level section. The cross-section notes contain information only at the center line and at the two slope stakes. The area of the cross section is the sum of the areas of four triangles labeled I, II, III, and IV. The areas are

$$A_{\mathrm{I}} = \frac{1}{2} \times \frac{b}{2} \times f_l; \quad A_{\mathrm{II}} = \frac{1}{2} f \times d_l; \quad A_{\mathrm{III}} = \frac{1}{2} f \times d_r; \quad A_{\mathrm{IV}} = \frac{1}{2} \times \frac{b}{2} \times f_r$$

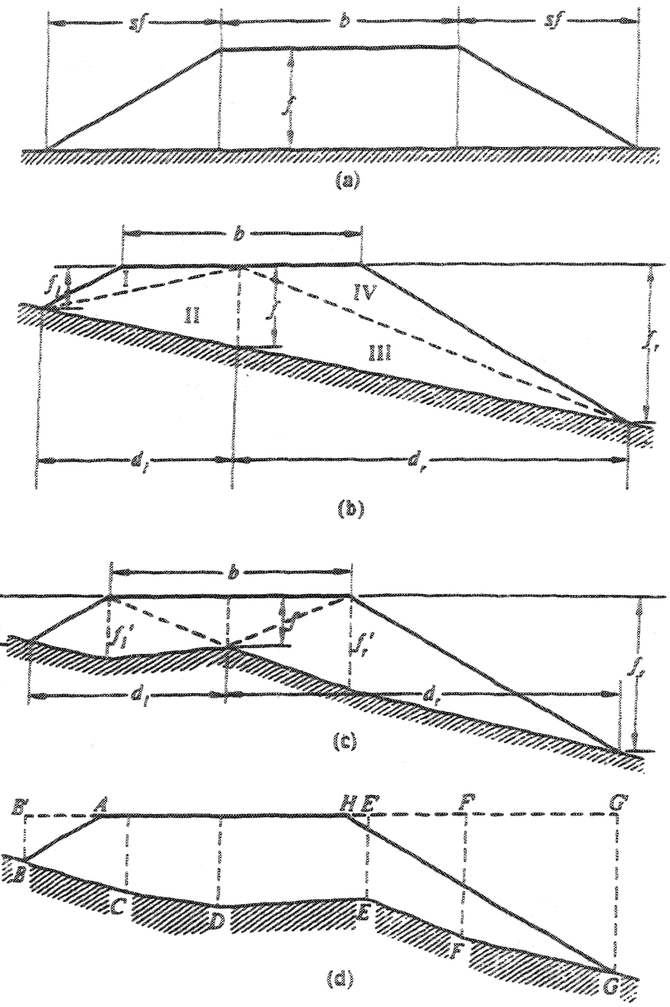

FIGURE 17-8 Types of cross sections. (a) Level section. (b) Three-level section. (c) Five-level section. (d) Irregular section.

Combining the areas of the four triangles gives

$$A = \frac{b}{4}(f_l + f_r) + \frac{f}{2}(d_l + d_r) \qquad (17\text{-}$$

in which f_l and f_r are the fill at left and right slope stakes, respectively; d_l and d_r ar the distances to left and right slope stakes, respectively; and f is the center fill. the section is in cut, the symbol c is substituted for f.

A five-level section is shown in view (c). The fill is determined at points mea sured out a distance equal to one-half roadbed width on both sides of the cente line. The area of this class of section is

$$A = \tfrac{1}{2}(f_l' d_l + f b + f_r' d_r) \qquad (17\text{-}$$

in which f'_l and f'_r are the fill at the left and right edge of the roadbed, respectively; and d_l and d_r are the distances to left and right slope stakes, respectively.

The cross section shown in Fig. 17-8(d) is called an *irregular section* in which the ground surface is so irregular that a three- or five-level section would not give enough information to obtain an accurate determination of the area. The area of the section may be obtained by computing the areas of the trapezoids forming the figure $B'BCDEFGG'$ and subtracting from their sum the areas of the two triangles ABB' and GHG'.

Another method, which can also be applied to any of the preceding figures, is to consider the cross section as a traverse. The field notes provide the coordinates of the corners with respect to the finished grade and the center line as coordinate axes, the horizontal distances from the center line being the X coordinates with due regard for algebraic sign, and the vertical cuts or fills being the Y coordinates. For an eight-sided traverse for example, the area can be expressed by one of the following equations, which are like Eq. (8-25) or (8-26) in Section 8-21:

$$\text{area} = \tfrac{1}{2}[X_1(Y_2 - Y_8) + X_2(Y_3 - Y_1) + X_3(Y_4 - Y_2) + \ldots]$$
$$\text{area} = \tfrac{1}{2}[Y_1(X_2 - X_8) + Y_2(X_3 - X_1) + Y_3(X_4 - X_2) + \ldots]$$

Since for a cross section in earthwork the Y coordinates of two of the points are zero, the computations will be shortened if the second equation is used. An application of this equation to the area in Fig. 17-8(d) is given in Table 17-1, in which the roadbed width is 24 ft.

17-9 Volume by Average End Areas

According to the end-area formula, the volume, in cubic feet, between two cross sections having areas A_0 and A_1 is

$$V_e = \tfrac{1}{2}(A_0 + A_1)L \tag{17-8}$$

TABLE 17-1 Computations for Area of Cross Section

A	B	C	D	E	F	G	H
$\frac{F0.0}{-12.0}$	$\frac{F5.2}{-19.8}$	$\frac{F6.8}{-10.0}$	$\frac{F7.2}{0}$	$\frac{F6.1}{+15.0}$	$\frac{F7.4}{+20.0}$	$\frac{F9.6}{+26.4}$	$\frac{F0.0}{+12.0}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

$Y_i(X_{i+1} - X_{i-1})$		= double area
0.0(− 19.8 − 12.0)	=	0.0
5.2(− 10.0 + 12.0)	=	+ 10.4
6.8(0.0 + 19.8)	=	+ 134.6
7.2(+ 15.0 + 10.0)	=	+ 180.0
6.1(+ 20.0 − 0.0)	=	+ 122.0
7.4(+ 26.4 − 15.0)	=	+ 84.4
9.6(+ 12.0 − 20.0)	=	− 76.8
0.0(− 12.0 − 26.4)	=	+ 0.0
		2)454.6
	Area =	227.3 ft²

in which L is the distance between the sections. The volume in cubic yards, is

$$V_e = \frac{1}{2}(A_0 + A_1)\frac{L}{27} = \frac{L}{54}(A_0 + A_1) \tag{17-9}$$

Although this relationship is not an exact one when applied to many earthwork sections, it is the one most commonly used because of the ease of its application and because of the fact that the computed volumes are generally too great and thus the error is in the favor of the contractor.

An example of the calculation of earthwork volumes by the average end-area formula for cut sections is given in Table 17-2. Note that the section at station 36 + 00 is three-level, and at station 37 + 00 is five-level. The roadbed is 36 ft, and side slopes are 2 : 1.

17-10 Volume by Prismoidal Formula

When ~~the more~~ exact volume must be known, it can be calculated by means of the prismoidal formula

$$V_p = \frac{L}{6}(A_0 + 4M + A_1) \tag{17-10}$$

in which M is the area of the middle section and V_p is the volume, in cubic feet. In general, M will *not* be the mean of the two end areas. It can be shown that this formula is correct for determining the volumes of prisms, pyramids, wedges, and prismoids that have triangular end sections and sides that are warped surfaces. Since the earthwork solids are included in this group, except for slight irregularities of the ground, the prismoidal formula gives very nearly the correct volume of earthwork. The error in the use of the end-area formula arises chiefly from the fact that in its application the volume of a pyramid is considered to be one-half the product of the base and the altitude, whereas the actual volume is one-third the product of those quantities.

The area of the middle section can be obtained by taking intermediate cross sections on the ground or when the same number of points have been taken on adjacent sections, by computing the area of a section that has dimensions equal to

TABLE 17-2 Computation of Volume of Earthwork by Average End-Area Formula

Station	Cross-Section Notes	Area
36 + 00	$\frac{C11.7}{41.4}$ $\frac{C7.4}{0.0}$ $\frac{C4.1}{26.2}$	$\frac{36}{4}(11.7 + 4.1) + \frac{7.4}{2}(41.4 + 26.2) = 392.3\ \text{ft}^2$
37 + 00	$\frac{C12.4}{42.8}$ $\frac{C9.1}{18.0}$ $\frac{C7.6}{0.0}$ $\frac{C6.3}{18.0}$ $\frac{C2.9}{23.8}$	$\frac{1}{2}(9.1 \times 42.8 + 7.6 \times 36 + 6.3 \times 23.8) = 406.5\ \text{ft}^2$
$V = \frac{100}{54}(392.3 + 406.5) = 1479\ \text{yd}^3$		